Reducing [normalizing, harmonizing, …] one observer’s data to another observer’s data can be described as a ‘mapping’ of the two datasets: \( \text{Obs } A = \text{Mapping (Obs B)} \). Svalgaard & Schatten (2016; S&S) showed that for annual averages their mapping function with high accuracy could be described as the slope of a simple linear regression going through the origin [Hoyt & Schatten (1998; H&S) used the, almost, equivalent simple ratio of all simultaneous observations]. Usoskin et al. (2016; UEA, doi: 10.1007/s11207-015-0838-1) argue that the S&S and H&S mappings were invalid. I’ll here argue that if two mappings consistently give the same results, either both mappings are valid or both are invalid.

In email (2016-07-15) Usoskin said Using the "mapping function" we get the annual Wolf's GN, reduced to Wolfer GN, divided by Wolfer, as 0.98, for 1881". This cleverly hides the real conversion factor, so I asked for clarification:

“Wolf observed the number of groups on certain days in 1881, now take you mapping and calculate the number Wolfer should have seen. Then add up the groups for Wolf and Wolfer for the year. Those are the relevant numbers that are of interest.”

The reply came back as “Wolf's non-corrected annual GN for 1881 was 2.84. Wolf-corrected-to-Wolfer (or what Wolfer should have seen according to Wolf and our daily correction matrix) 4.55”. I think he meant Wolf-corrected-to-Wolfer.

The ratio 4.55/2.84 is 1.602 which is closely the same as we get using the simple annual means. It turns out that the UEA methodology (mapping) gives the same result as S&S for comparison between Wolf and Wolfer, throughout. Thus, if the UEA mapping [methodology] is valid, so is the S&S’s because they give the same result when used on the same data. The UEA mapping is thus no worse than the S&S mapping.

So, here is what we get if we compare S&S and UEA since 1861 [where the data coverage is good]:

![Comparison of Group Counts Scaled by Different Methods](image-url)
In addition to the published S&S series (blue) I have added a ‘mini-backbone’ based on the observers shown (black: Tacchini, etc), and, for comparison UEA (red). All these series are normalized to Wolfer [who is the current World Standard observer, per SILSO]. We note that the agreement between UEA and S&S [for the period, 1876-1899, since we have Wolfer data; green box] is excellent: both mappings give very similar results. A possible exception is for very low values of activity. It is a pity that UEA did not extend their analysis into the modern period [1900 to the present], but have chosen to make their series inhomogeneous by appending a different dataset [based on Lockwood et al (2014)].

It is of interest to compare directly the UEA and S&S data:

There is a slight non-linearity [red curve] which we can approximate by a 3rd-degree polynomial. Comparison between the UEA values and actual observations by Wolfer shows that the UEA values are underestimated by almost a factor of two near sunspot minima.

<table>
<thead>
<tr>
<th>Year</th>
<th>UEA</th>
<th>Wolfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1876.5</td>
<td>0.50</td>
<td>0.93</td>
</tr>
<tr>
<td>1877.5</td>
<td>0.44</td>
<td>0.84</td>
</tr>
<tr>
<td>1878.5</td>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td>1879.5</td>
<td>0.31</td>
<td>0.55</td>
</tr>
<tr>
<td>1887.5</td>
<td>0.75</td>
<td>1.48</td>
</tr>
<tr>
<td>1888.5</td>
<td>0.45</td>
<td>0.80</td>
</tr>
<tr>
<td>1889.5</td>
<td>0.31</td>
<td>0.63</td>
</tr>
<tr>
<td>1890.5</td>
<td>0.40</td>
<td>0.85</td>
</tr>
<tr>
<td>Average</td>
<td>0.42</td>
<td>0.79</td>
</tr>
</tbody>
</table>

Had the UEA included modern data, we could have checked if this discrepancy is a general feature of their mapping or just statistical noise. Adjusting the UEA values using the non-linear fit, we can plot the result (green), and notice good agreement back to ~1865, with some divergence before that, which we ascribe to using different data. On the other hand for medium and high solar activity the agreement is very good.
For completeness we also included the H&S group number (purple) showing the now well-established divergence before ~1885. UEA matches S&S much better than it does H&S.

**Extending the Record past 1900**

UEA extended their series using the sunspot area record(s). We shall go further and extend the comparison with several other solar indices. We start with the observed sunspot area record, SA (not corrected for foreshortening), based on Balmaceda’s 2009 series of essentially the RGO observations. We base the mapping from sunspot area to group number on the RGO data for 1874-1976:

![Mapping Sunspot Areas to Group Numbers](image)

The relationship is non-linear as shown. We note that basing the mapping solely on the Wolfer Backbone (blue circles) leads to the same result. We use the observed (that is: projected) sunspot area in millionth of the solar disk, as both that area and the group number refer to direct observations of features on the disk.

The sunspot area data after RGO stopped observing (1976) are less secure as there are large differences between observers. Balmaceda et al. (2009) have attempted to harmonize the data series, but the scatter and uncertainties are larger than desirable.

The solar microwave flux, F, at $\lambda = 10.7$ cm has been observed since 1947 and is often thought of as a good proxy for solar activity. Its mapping to the Group Number is also non-linear. We fit two [admittedly arbitrary, but well-fitting] functions to the flux values and use the average of the function values as our mapping of the flux to the group number.
The XUV/EUV flux from the Sun maintains a dynamo current in the ionospheric E-region causing an easily observed diurnal variation [discovered in 1722] at ground level. The range of the variation, $r_Y$, of the East-West ($Y$-) component is a good measure of the XUV/EUV flux and can be mapped to the Group Number:

![Mapping F10.7 Microwave Flux to Sunspot Group Number](image)

$$GN = - \frac{F^2}{6300} + 0.1171F - 6.72$$
$$R^2 = 0.986$$

![Mapping $r_Y$ to Group Numbers](image)

$$GN = 0.44r_Y - 14.21$$
$$R^2 = 0.962$$
Another good proxy for the UV flux is the Mg II-index. In the core of the Mg II Fraunhofer line, the H and K chromospheric emissions are observable. The Mg II-index is the ratio between the chromospheric emission and the solar continuum. As a ratio, the index is quite insensitive to calibration and degradation; a composite can be formed from several spacecraft instruments (the Bremen-index). We have reliable data since 1979 and find the following (nicely linear!) mapping function:

A recent HMI-Nugget [http://hmi.stanford.edu/hminuggets/?p=1510] shows that the F10.7 Microwave Flux Matches the Total Disk Unsigned Magnetic Flux from MDI (SOHO) and HMI (SDO):
We can also determine the mapping function for the magnetic flux to the group number:

Putting all the mappings together we get:

All curves track each other well to within about one group count. If the mappings were incorrect or the Group Numbers were not representative of underlying real solar activity, there would be systematic disagreements. None are evident. We now form a simple composite by averaging all the curves. Trying to assign a weight to each curve and forming a weighted average is probably going beyond what the data warrant.

The three scaled indicators, rY*, SA*, and GN* span the whole time interval from 1874 to the present. The indicators are independent and the mappings are not just simple proportionalities, but do take into account the non-linearities inherent in the data.
H&S did not employ ‘daisy chaining’ when they had RGO group counts to compare with in the 20th century. For years before ~1910 there does seem to be a drift in the number of groups reported by RGO compared with what other regular, long-time, and seasoned observers reported:

The reason for this is not known; perhaps we are seeing a learning curve, as apportioning spots to groups is hard [contrary to the oft repeated statement that counting groups is easier]. Determining the area of all the spots is easy, as it is just a mechanical procedure counting dark pixels. In fact, we showed above that the mapping of SA to GN is quite uniform with no indication of depicting a changing or unstable population. We take the agreements between the mapped values of the various solar indices as indicative of a validation of both the underlying data and of the methodologies employed.

It seems that a small ‘cottage industry’ has sprung up of claiming that the H&S Sunspot Group Number is some kind of Gold standard against which any and all proposed revisions have to be judged. So it is of interest to compare H&S with the composite we have developed here. Because of the likely drift of RGO takes place mostly before about 1910, we concentrate on comparing over the interval 1910-1995:
Applying this mapping function it is immediately clear that the agreement between H&S and our composite is extraordinarily good ($R^2 = 0.993$) for the interval 1910-1995:

The green curve at the bottom of the Figure is the standard deviation of the values that go into our composite. The average standard deviation is only 0.34 groups with no long-term trend over 1874-2016. So, in spite of all the objections, hand wringing, and acrimony, the methods used by UEA, H&S, and S&S all give the same results when the underlying data are good enough and the excessive daisy chaining employed by H&S backwards in time from the RGO data is avoided.