Abstract. The large decrease in the horizontal component of the earth’s field during the main phase of magnetic storms has been ascribed to the formation or enhancement of a geomagnetic ring current. In this paper we discuss the motions of particles trapped in the earth’s dipole field and the resulting ring current. These calculations deal only with a steady state, though during storms the state is changing. The general equations for the current intensity, to obtain the total current and the magnetic field at the earth’s center, are applied to the outer radiation belt (V₁) and to a special ‘model’ belt V₂. This V₂ belt has a particular type of pitch-angle distribution and a number-intensity distribution of Gaussian type along an equatorial radius. The results are considered in connection with magnetic records for several storms and with satellite data. We infer that, during magnetic disturbance, protons of energy of the order of a few hundred kev are intermittently captured between 5 and 8 earth radii and that they produce a transient belt V₂. The variety of development of the ring current from one storm to another may be connected with irregularities in the distribution of particles in the solar stream, which may contain tangled magnetic fields.

1. Introduction

Two major average features of geomagnetic storms are an initial increase in the horizontal magnetic force at the earth’s surface and a subsequent larger and more prolonged decrease. Many years ago Schmidt [1917] ascribed the decrease to the influence of a westward electric current that encircles the earth; this is now generally called the (geomagnetic) ring current. He concluded that it must wax and wane as magnetic disturbance increases or decreases. From a study of the monthly means of the horizontal component (at Potsdam) he suggested that the current never dies away completely but is a permanent companion of the earth.

Like most students of geomagnetic disturbance, Schmidt attributed magnetic storms to the influence of corpuscular streams or clouds from the sun. The clear implication of his conclusions was that some of the solar matter remains near the earth for a considerable period. In the phraseology now common, the solar material is partly trapped by the earth’s magnetic field, and one consequence is the enhancement of the ring current and its geomagnetic effects. Schmidt did not consider the scale or the location of the current.

Chapman and Ferraro [1931a, 1931b, 1933, 1940] and Ferraro [1952] attempted to infer by mathematical deduction the consequences of the
impact of a neutral ionized solar stream upon the earth. Their theory discussed an idealized 'model,' but in more than one respect it left out significant features of the actual situation. Their inferences, though imperfect and incomplete, indicate the essential mechanism of the first phase of magnetic storms, and the scale of the penetration of the solar gas. The deflection of the gas particles by the geomagnetic field, by which much of the gas is continuously reflected or scattered away from the earth, produces the first phase of the storm. It seemed clear that some of the gas found its way into the earth's atmosphere in high latitudes (and there produced auroras), and also that some was retained in the field for a time in the form of a ring current. They were unable to explain or delineate the motions of these two subgroups of the solar particles. However, postulating (like Schmidt) the existence of the ring current, on the basis of the geomagnetic evidence, they discussed the equilibrium, stability, and decay of the current. The model current ring in their discussion was toroidal, with protons and electrons simply circulating around the geomagnetic axis, with slightly different speeds, the motion of the protons, at least, being westward. According to Alfvén [1958] this type of ring current has been shown by Herlofson to be unstable.

Singer [1957] proposed a different concept of the trapped component of the solar gas, based on the work of Störmer and Alfvén. Instead of a toroidal form, and simple circular motion for most of the particles of the gas, he saw that the gas would have the form and motions not long afterward indicated by satellite and cosmic rocket exploration [Van Allen and Frank, 1959; Vernov, Chudakov, Vokulov, and Logachev, 1959]. The particles oscillate rapidly to and fro between mirror points in fairly high northern and southern latitudes. At the same time they circle round the magnetic field lines, and also drift round the earth—the protons westward, the electrons eastward. Singer concluded that these motions necessarily correspond to a ring current around the earth. Later the ring current and its field were discussed by Dessler and Parker [1959] and by Akasofu [1960].

Meanwhile satellite observations bearing on these problems have been made during magnetically quiet and disturbed periods. The radiation belts have been much observed, and some magnetic measurements have been made in them, by the U. S. S. R. Mechina and the U. S. Explorer VI. The Mechina found magnetic deviations that indicated a ring current in the outer Van Allen belt (here the inner and outer belts will be denoted by $V_1$ and $V_2$). The Explorer VI found evidence of a ring current much farther out, in a region beyond 5 earth radii, where we have inferred its presence from auroral and magnetic data. We denote it by $V_s$.

In sections 1 to 5 we discuss the motions of trapped particles in the region of the radiation belts, and the associated currents. In section 6 we analyze several magnetic storms that occurred during the IGY and IGC 1959, and discuss the development of the ring current during these storms.

1.1 Notation (General)

The following general notation is used:

- $a$ = the earth's radius.
- $r$ = $r_0$ = the radial distance from the earth's center $O$ to a point $P$.
- $m$ = the mass of a trapped particle.
- $e$ = its charge, in esu.
- $w$ = its velocity.
- $w$ = its speed.
- $E$ = its energy, in kev, namely, $\frac{1}{2}mw^2 / 1.602 \times 10^{-9}$.
- $n$ = the number density of the particles.
- $H$ = the magnetic vector, of magnitude $H$.
- $h$ = the unit vector along $H$, so that $H = Hh$.
- $p_m$ = the magnetic pressure, namely $H^2 / 8\pi$.
- $\theta$ = the angle, called the pitch angle, between $w$ and $H$.
- $w_\perp$ = the component of $w$ along $H$, so that $w_\perp = w \cos \theta$.
- $w_\parallel$ = the component of $w$ normal to $H$, so that $w_\parallel = w \sin \theta$.
- $p_\parallel$ = the pressure of the gas along $H$; it is the sum of $mw_\parallel^2$ over all the particles per unit volume.
- $p_\perp$ = the pressure of the gas normal to $H$; it is the sum of $\frac{1}{2}mw_\perp^2$ over all the particles per unit volume.
- $\phi, \Phi$ = the geomagnetic latitude and longitude.
- $r_s = r_0$ = the distance from $O$ to the point $P_s$ at which the dipole line of force through a point $P$ given by $r$ (or $r_0$), $\phi, \Phi$ crosses the equatorial plane of the dipole.
1.2. Geometrical Formulas Relative to a Dipole Field

Here and later in this paper the terms axis, latitude, longitude, equator, equatorial or meridian plane will refer to a dipole field, namely, in general, to the geomagnetic dipole field. The subscript e will signify reference to the equatorial plane.

The position of any point \( P \) will be specified by the usual spherical polar coordinates \( r, \phi, \Phi \), or by \( r_e, \phi, \Phi \), where \( r_e \) denotes the radial distance of the related point \( P_e \), where the dipole line of force through \( P \) cuts the equatorial plane. This line will often be referred to as the line (of force) \( r_e \) or \( f_e \), if \( r_e = f_e/a \). Its equation is

\[
r = r_e \cos^2 \phi \quad \text{or} \quad f = f_e \cos^2 \phi
\]  

if \( f = r/a \).

With each point \( P \) we associate two unit vectors \( j, k \), which with \( h \) form a right-handed orthogonal unit vector triad \( h, j, k \). The definitions of \( j \) and \( k \) are:

\( k \) -- the eastward unit vector normal to the meridian plane through \( P \);

\( j \) -- the unit vector \( k \times h \); it is outward from the origin, but not radially outward.

Let \( P' \) be a point adjacent to \( P \) with coordinates differing by \( \delta r_e, \delta \phi, \delta \Phi \) from those of \( P \), namely, \( r_e, \phi, \Phi \). The vector element \( PP' \) or \( ds \) can be specified as follows relative to the vector triad \( h, j, k \):

\[ ds = h h_1 d\phi + j h_2 \delta r_e + k h_3 d\Phi \]

Here \( h_1 \), \( h_2 \), and \( h_3 \) can be thus specified in terms of \( r_e, \phi, \Phi \):

\[
h_1 = r_e(1 + 3 \sin^2 \phi)^{1/2} \cos \phi
\]

\[
h_2 = \cos^3 \phi/(1 + 3 \sin^2 \phi)^{1/2}
\]

\[
h_3 = r_e \cos^3 \phi
\]

Hence an element of area \( dS_3 \) in the meridian plane through \( P \), bounded by the lines of force \( r_e, \delta r_e + \delta r_e \) and the radii with latitudes \( \phi, \phi + d\phi \), is given by

\[
dS_3 = h_1 h_2 \delta r_e d\phi = r_e \cos^4 \phi \delta r_e d\phi
\]

This surface element is normal to \( k \). The surface element \( dS_1 \) normal to \( h \) is similarly given by

\[
dS_1 = h_2 h_3 \delta r_e d\Phi
\]

or

\[
[r_e \cos^6 \phi/(1 + 3 \sin^2 \phi)^{1/2}] \, dr_e d\Phi
\]

Since the tubes of magnetic force are of constant strength, it follows that

\[
Hh_3 h_3 = (Hh_2 h_3)_e
\]

Hence

\[
H_e/H = h_2 \cos^3 \phi
\]

The field intensity \( H_e \) in the equatorial plane is given by

\[
H_e = a^3 H_0/r_e^3
\]

Here \( H_0 \) denotes the value of \( H_e \) at the equator at the earth's surface. It is taken to be 0.32 gauss (=3.2 \times 10^7 \gamma).

A volume element \( dV \) at \( P \) can similarly be expressed by

\[
dV = h_1 h_2 h_3 \delta r_e \delta \phi \delta \Phi
\]

or

\[
[r_e \cos^6 \phi/(1 + 3 \sin^2 \phi)^{1/2}] \, dr_e d\Phi
\]

The gradient of a scalar function \( Q \) in our coordinate system is given by

\[
\nabla Q = h \frac{1}{h_1} \frac{\partial Q}{\partial \phi} + j \frac{1}{h_2} \frac{\partial Q}{\partial \delta r_e} + k \frac{1}{h_3} \frac{\partial Q}{\partial \delta \Phi}
\]

If the system is axially symmetric, the last term in (9) is zero, and then

\[
h \times \nabla Q = \frac{1}{h_3} \frac{\partial Q}{\delta \Phi} k
\]

Note also that

\[
(H \cdot \nabla)H = -H^2 j/R_e
\]

where \( R_e \) denotes the length of the radius of curvature of the line \( r_e \) at \( P \); \( R_e \) is given by

\[
\frac{1}{R_e} = \frac{3(1 + \sin^2 \phi)}{r_e \cos \phi (1 + 3 \sin^2 \phi)^{1/2}}
\]

2. THE MOTION OF CHARGED PARTICLES IN THE EARTH'S DIPOLE FIELD

As was shown by Alfvén [1950], the motion of a charged particle in a magnetic field can in certain circumstances be analyzed into (a) the motion of a 'guiding center' associated with the particle (this motion being partly along and partly across the lines of force), and (b) nearly
circular motion around the lines of force, relative to the guiding center. The motion of the guiding center across the lines of force is often called a drift. The guiding-center approximation is valid (i) when the average radius \( R \) of the circular motion is much less than the scale length of the system considered, and (ii) when the period \( T \) of the circular motion is much less than the other scale times associated with the phenomenon. Chapman [1961] has defined the scale length and time. In the earth's undisturbed dipole field the scale length is of the order of 8000 km, and the scale time may be of the order of \( 10^4 \) seconds in the ring current problem. Table I gives values of \( R \) and \( T \) for typical particles in the radiation belts. Clearly both conditions (i) and (ii) are satisfied for such particles.

2.1. The Three Motions in a Dipole Field

(a) Oscillation between the two mirror points. In the rapid oscillations of a trapped particle between the 'mirror' points \( M \) and \( M' \) in the northern and southern hemispheres, the magnetic moment \( \mu \) associated with the particle and its motion is invariant; \( \mu \) is given [Alfvén, 1950, p. 20] by

\[
\mu = \frac{1}{2} mw^2/H = \frac{1}{2} mw^2/H_m
\]

Here \( H_m \) denotes the value of \( H \) at the mirror point \( M \), where \( w_s = 0, w_e = w \). Thus, at \( M \),

\[
H_m = \left( \frac{w}{w_m} \right)^2 H_s = H_s / \sin^2 \theta_s
\]

The latitude \( \phi_m \) of \( M \) depends only on \( \theta_s \):

\[
\sin^2 \theta_s = \left( \cos \phi_m / (1 + 3 \sin^2 \phi_m) \right)^{1/2}
\]

The height \( h_m \) of \( M \) above the ground is given by the equations

\[
h_m = r_m - a
\]

\[
\sin^2 \theta_s = \left( r_m / r_s \right)^3 / (4 - 3r_m / r_s)^{1/2}
\]

Figure 1 has been constructed from equations 16 and 17 to show how \( h_m, r_s, \) and \( \theta_s \) are related. The smaller the distance \( r_s \), the larger the range

**TABLE 1.** Numerical Particulars for Electrons and Protons of Various Energies That Cross the Equatorial Plane with Pitch Angle \( \theta_s = \sin^{-1} 0.1 \) and Are Associated with the Lines of Force \( r_s = 6a \) of the Earth's Dipole Field

In this table a number expressed as \( X^y \) signifies \( X \times 10^y \).

<table>
<thead>
<tr>
<th>( R_M )</th>
<th>( w_s )</th>
<th>( E_s )</th>
<th>( C_{st} )</th>
<th>( R_s )</th>
<th>( T_s )</th>
<th>( \delta \lambda )</th>
<th>( 360^\circ / \delta \lambda )</th>
<th>( T_{ov} )</th>
<th>( T_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^e)</td>
<td>1.76(^{\circ})</td>
<td>0.879</td>
<td>9.00(^{\circ})</td>
<td>6.76(^{\circ})</td>
<td>2.42(^{-4})</td>
<td>0.0055</td>
<td>65455</td>
<td>1 09(^{-4})</td>
<td>764 days</td>
</tr>
<tr>
<td>3(^e)</td>
<td>5.20(^{\circ})</td>
<td>7.85</td>
<td>5.20(^{\circ})</td>
<td>2.03(^{-1})</td>
<td>2.46(^{-4})</td>
<td>0.0185</td>
<td>19459</td>
<td>36.7</td>
<td>8.3 days</td>
</tr>
<tr>
<td>6(^e)</td>
<td>9.95(^{\circ})</td>
<td>30.7</td>
<td>3.67(^{\circ})</td>
<td>4.06(^{-1})</td>
<td>2.57(^{-4})</td>
<td>0.0356</td>
<td>10121</td>
<td>1.92</td>
<td>5.4 hours</td>
</tr>
<tr>
<td>1(^i)</td>
<td>1.52(^{\circ})</td>
<td>81.4</td>
<td>2.85(^{\circ})</td>
<td>6.76(^{-1})</td>
<td>2.81(^{-4})</td>
<td>0.0578</td>
<td>6233</td>
<td>1.27</td>
<td>2.2 hours</td>
</tr>
<tr>
<td>5(^i)</td>
<td>2.84(^{\circ})</td>
<td>1.07(^{\circ})</td>
<td>1.27(^{\circ})</td>
<td>3.38</td>
<td>7.48(^{-4})</td>
<td>0.2845</td>
<td>1265</td>
<td>0.673</td>
<td>14.2 minutes</td>
</tr>
<tr>
<td>1(^i)</td>
<td>2.96(^{\circ})</td>
<td>2.53(^{\circ})</td>
<td>9.00(^{\circ})</td>
<td>6.76</td>
<td>1.44(^{-3})</td>
<td>0.5673</td>
<td>635</td>
<td>0.645</td>
<td>6.8 minutes</td>
</tr>
<tr>
<td>5(^k)</td>
<td>4.79(^{\circ})</td>
<td>2.0</td>
<td>1.27(^{\circ})</td>
<td>3.38</td>
<td>0.433</td>
<td>0.2845</td>
<td>1265</td>
<td>3.99(^{2})</td>
<td>6.0 days</td>
</tr>
<tr>
<td>1(^k)</td>
<td>9.58(^{\circ})</td>
<td>4.79</td>
<td>9.00(^{\circ})</td>
<td>6.76</td>
<td>0.433</td>
<td>0.5673</td>
<td>635</td>
<td>1.99(^{2})</td>
<td>35 hours</td>
</tr>
<tr>
<td>3(^k)</td>
<td>2.87(^{\circ})</td>
<td>45.1</td>
<td>5.20(^{\circ})</td>
<td>2.03(^{\circ})</td>
<td>0.433</td>
<td>1.712</td>
<td>210</td>
<td>66.6</td>
<td>3.9 hours</td>
</tr>
<tr>
<td>6(^k)</td>
<td>5.75(^{\circ})</td>
<td>172</td>
<td>3.67(^{\circ})</td>
<td>4.06(^{\circ})</td>
<td>0.433</td>
<td>3.429</td>
<td>105</td>
<td>33.2</td>
<td>58 minutes</td>
</tr>
<tr>
<td>5(^k)</td>
<td>9.57(^{\circ})</td>
<td>479</td>
<td>2.85(^{\circ})</td>
<td>6.76(^{\circ})</td>
<td>0.433</td>
<td>5.688</td>
<td>63.3</td>
<td>20.0</td>
<td>21 minutes</td>
</tr>
<tr>
<td>1(^k)</td>
<td>9.13(^{\circ})</td>
<td>4.79(^{\circ})</td>
<td>9.00(^{\circ})</td>
<td>6.44(^{\circ})</td>
<td>0.433</td>
<td>57.06</td>
<td>6.3</td>
<td>2.09</td>
<td>16 seconds</td>
</tr>
</tbody>
</table>

* The relativistic correction is made for high-energy electrons in calculating \( R \) and \( T \).
RING CURRENT, GEOMAGNETIC DISTURBANCE

Fig. 1. The graphs show for what combinations of $f_s$ and $\theta_s$ the mirror points have heights of 100, 300, 500, and 1000 km above the ground; $f_s = r_s/a$, and $r_s$ is the distance at which a particle crosses the equatorial plane with pitch angle $\theta_s$.

of pitch angle for which the particles can penetrate the atmosphere.

The time $T_0$ required for one complete oscillation between the two mirror points (from $P_s$ to $M$, thence through $P_s$ to $M'$ and back to $P_s$) is given by

$$T_0 = 4l/w$$

(18)

Here $l$ denotes the arc length of the spiral path of the particle from $P_s$ to $M$ or $M'$. Its value has been calculated by Wentworth, MacDonald, and Singer [1959, see their Fig. 2]. For the case $w_s/w = \sin \theta_s = 0.1$, $l$ is approximately given by

$$l = 1.25r_s$$

(19)

In calculating the data of Table 1, the distance $r_s$ is taken to be $6a$. The field intensity $H_s$ at $6a$ is about 148 gammas. Thus equation 14 shows that a particle for which $\sin \theta_s = 0.1$ will be reflected at a point $M$ where the field intensity on the line of force crossing the equatorial plane at 6 earth radii is 14,800 gammas. In this case $M$ is 3190 km above the ground. The corresponding mirror latitude $\phi_0$ is approximately $60^\circ$. For such a particle, according to equation 19,

$$l = 7.50a$$

(20)

and

$$T_0 = 30a/w = 1.91 \times 10^{10}/w \text{ sec}$$

(21)

if $w$ is expressed in centimeters per second.

(b) The slow east-west drift motion. During the oscillations between the mirror points, the protons drift westward and the electrons drift eastward. The guiding centers move on a surface of revolution containing the lines of force $r_s$.

The drift motion results from two causes: (a) the inhomogeneity of the earth's field, and (b) the centrifugal force associated with the motion along the curves lines of force. The corresponding parts $u_1$ and $u_2$ of the drift velocities are respectively given [cf. Parker, 1957] by the general formulas

$$u_1 = (\frac{1}{2} m w_e^2 c/eH^4) H \times \nabla H^2/2$$

(22)

$$u_2 = (m w_e^2 c/eH^4) H \times (H \cdot \nabla) H$$

(23)

For the earth's dipole field, these two equations may be transformed to

$$u_1 = -k(\frac{mc}{eH})(\frac{1}{2} w_e^2 / R_M)$$

(24)

$$u_2 = -k(\frac{mc}{eH})(w_e^2 / R_M)$$

(25)

and $R_M$ is given by equation 12.

During each complete oscillation between the two mirror points the particle has a longitudinal displacement $\delta \lambda$ resulting from the above two drift motions. It is given [Alfvén, 1950, p. 28] by

$$\delta \lambda = 4(r_s/C_s)^2 I_1(\phi_0)$$

(26)

Here $C_s$ denotes Störmer's unit length [Störmer, 1955, p. 217], given by

$$C_s = (Mc/cm)/(M/R_M)$$

(27)

where $M$ denotes the dipole moment of the earth $(8.1 \times 10^{10}$ gauss cm), and

$$R_M = mc/e$$

(28)

$R_M$ is called the magnetic rigidity of the charged particle. This length $C_s$ was introduced by Störmer in his calculations of the paths of single particles entering the earth's magnetic field from outside, or trapped in the field. Also $I_1$ is the integral

$$I_1(\phi_0) = \int_0^\phi \frac{h_0}{R_M} \left(1 - \frac{1/2}{H/H_M} \right)^{1/2} d\phi$$

(29)

The integral $I_1(\phi_0)$, an angle, has been eval-
Fig. 2. The pitch-angle distribution $F \propto \sin^{\alpha - 1} \theta$ for $\alpha = 2, 0, -0.9$. The direction $\theta = 0$ is parallel to H. The curves are normalized to give the same mean density in each case. The deviation from the isotropic distribution ($\alpha = 0$) is shown by a dotted area for $\alpha = 2$ and by a hatched area for $\alpha = -0.9$. For $\alpha > 0$ the large pitch angles are in excess; for $\alpha < 0$, the smaller pitch angles.

uated by Alfvén [1950, p. 29] for a series of values of the mirror latitudes $\phi_0$. When $\phi_0$ exceeds 45°, $I_1(\phi_0)$ is nearly independent of $\phi_0$, with the approximate value 76°. In that case the longitudinal displacement per complete oscillation is given by

$$d\lambda = 4 \times 76^\circ (r_s/C_s)^2$$

(30)

The number of oscillations made by the particle per revolution is $(360^\circ/d\lambda)$. The time $T_R$ required for one revolution is given by

$$T_R = (360^\circ/d\lambda)T_0$$

(31)

(c) Gyration. The radius $R$ and the period $T$ of the gyration of the particle are given by

$$R = cmw_0/eH$$
$$T = 2\pi cm/eH$$

(32) (33)

Table 1 illustrates these formulas numerically, for protons and electrons that cross the equatorial plane at $r_s = 6a$ with a pitch angle $\theta_s = \sin^{-1} 0.1$; the latitude of the mirror points for these particles is approximately 60°. Various energies are considered, corresponding to a series of values of the magnetic rigidity $B_M$ that are included in a table given by Störmer [1955, p. 294, Table 1].

For protons of energy greater than 10 Mev, the time of revolution $T_R$ becomes comparable with the time of oscillation $T_0$. The guiding-center approximation then ceases to be valid, and the radius of gyroratory motion becomes comparable with the radius of the earth.

3. Electric Currents in an Ionized Gas; General Formulas

All three motions (a), (b), and (c) discussed in section 2 may produce electric currents. The current associated with the oscillatory motion (a) of the charged particles, along H, is given by the formula appropriate when the magnetic field is absent, namely,

$$i = \Sigma e\nu$$

(34)

Here the summation includes both electrons and protons.

Parker [1957] gave expressions for the volume current densities $i_D$ and $i_C$ arising respectively from (b) and (c). For $i_D$ he gave the formula

$$i_D = en(u_1 + u_2) = \frac{e}{8\pi p_m} H \times$$

$$\{\partial(p_m/p_m) \nabla p_m + (p_1/p_m)(H \cdot \nabla)H/8\pi\}$$

(35)

(his equation 21), in accordance with (22) and
The part $i_L$, due to the gyration, is given by his equation 19,*

$$i_L = \frac{c}{8\pi p_m} \mathbf{H} \times \left\{ \nabla p_n - \frac{1}{2} (p_n/p_m) \nabla p_m \right\} - \frac{(p_n/p_m) (\mathbf{H} \cdot \nabla) \mathbf{H}}{8\pi} \right\}$$

(36)

The three current components (34), (35), and (36) correspond to the particle motions when there is no electric field. If conditions are changing, and an electric field $\mathbf{E}$ is present, the particles will have an additional velocity $\mathbf{v}$ given by

$$\mathbf{v} = c \mathbf{E} \times \mathbf{H} / H^2$$

(37)

Consequently there will then be a fourth contribution, $i_P$, to the current. This is given by 
Parker [1957] in the form (correcting a misprint in the original):

$$i_P = \left( \frac{mc}{H^2} \right) \mathbf{H} \times (d\mathbf{v}/dt)$$

(38)

As in (35) and (36), this is perpendicular to $\mathbf{H}$. Thus the total volume current $i$ in an ionized gas is the combination of (34) along $\mathbf{H}$ and the sum $i$ of the above three currents perpendicular to $\mathbf{H}$:

$$i = i_n + i_L + i_P = \left( \frac{c}{8\pi p_m} \right) \mathbf{H} \cdot \nabla p_m + \nabla p_m \right\}$$

(39)

4. The Steady Ring Current in a Dipole Field

We now limit our discussion to the special case of a steady unchanging system of particles, symmetrical round the axis, in the geomagnetic dipole field. Thus we omit the last term in (39), use equations 10 and 11, and obtain

$$i = -i_k$$

(40)

where

$$i = \left( \frac{c}{HR_c} \right) (p_n - p_m) - \left( \frac{c}{H h_2} \right) (\partial p_n/\partial r_n)$$

(41)

The negative sign in equation 40 signifies that a positive term in $i$ denotes a westward contribution to the current density $i$, and a negative term denotes an eastward contribution. For $R_c$ and $h_2$, see equations 12 and 3.

Both terms in (41) depend on the pressure of the gas formed by the particles. This in turn depends on the number and velocity distribution of the particles. Owing to the continual gyration of the particles around the lines of force, the velocity distribution function will depend only on the speed $w$ and the pitch angle $\theta$, but not on the azimuth of $w$ relative to the vector $\mathbf{h}$. Let $F(w, \theta) \, dw \, d\theta$ denote the number density of the particles whose speeds lie between $w$ and $w + dw$, and whose pitch angles lie between $\theta$ and $\theta + d\theta$. There will be such a function for each type of particle present. Thus

$$p_* = \sum \int \int mw^2 F(w, \theta) \, dw \, d\theta$$

$$= \sum \int \int \frac{1}{2} mw^2 F(w, \theta) \, dw \, d\theta$$

(42)

$$p_n = \sum \int \int \frac{1}{2} mw^2 F(w, \theta) \, dw \, d\theta$$

(43)

where the summation includes terms for each kind of particle, indicated by the addition of subscripts 1, 2, \ldots; $m, w, F$ will all have such subscripts in each term.

For the present we consider particles of one type only, with speeds between $w$ and $w + dw$. For brevity the symbols $\Sigma$ and $\int \cdots \, dw$ will be omitted from our equations. Expressions so abbreviated will be denoted by an asterisk, thus:

$$p_* = mw^2 \int_0^\infty F(w, \theta) \cos^2 \theta \, d\theta$$

$$p_n* = \frac{1}{2} mw^2 \int_0^\infty F(w, \theta) \sin^2 \theta \, d\theta$$

(44)

This equation gives only one 'item' of $p_*$ and $p_n$. To obtain the complete value from such an item expression we must integrate with respect to $w$ and sum over all the types of particle present. These conventions will be applied not only to $p_*$ and $p_n$ but also to all quantities derived from them or from $F(w, \theta)$. For example, the corresponding item of number density is given by

$$n* = \int_0^\infty F(w, \theta) \, d\theta$$

(45)
The function $F(w, \theta)$ will in general depend not only on the variables explicitly indicated in its notation but also on the position of the point $P$ to which it relates. Thus it is also a function of $r_\ast$ and $\phi$, but not of $\Phi$, according to our condition of symmetry about the dipole axis. Owing to the relation of the motion of the particles to the magnetic field, $F$ cannot be an arbitrary function of $r_\ast$ and $\phi$. Parker [1957] has discussed the relation between the form of $F$ as a function of position, for points along any line of force in a general magnetic field. He showed that the relation is especially simple if $F$ involves $\phi$ only by a factor $\sin \alpha \theta$. If $F$ is thus related to $\theta$ at one point of a line of force, it has the same relation at all points along the line. But in general the cofactor of $\sin \alpha \theta$ in $F(w, \theta)$ will vary along the line, proportionately to $H^{-1/1.5}$.

4.1 Special Pitch-Angle Distribution

Applying this general result to the dipole field here considered, the relation between the values of $n^*$ at a general point $P$ and at its associated equatorial point $P_e$ on the line of force $r_\ast$ is given by

$$n^*(r_\ast, \phi) = n^*(r_e)(H_e/H)^{1/2} \alpha$$  \hspace{1cm} (46)

For the last factor see equation 6.

The relation 45 between $n^*$ and $F$ when $F$ has this special form of dependence on $\theta$ is

$$F(w, \theta) = n^* A(\alpha) \sin \alpha \theta$$  \hspace{1cm} (47)

where

$$A(\alpha) = \Gamma(\alpha + 2)/2\alpha^2 \{\Gamma(\alpha + 1)\}^2$$  \hspace{1cm} (48)

The expression 47 applies to any point, such as $P$ and $P_e$; the appropriate values of $n^*$, related as in (46), must be used in each case.

Inserting the special expression given by (47) in (44) we have

$$\rho^*_w = 2mv^2 B(\alpha)n^*$$  \hspace{1cm} (49)

$$\rho^*_v = mw^2 \{1 - 4B(\alpha)\} n^*$$

where

$$B(\alpha) = (\alpha + 2)/(\alpha + 3)$$  \hspace{1cm} (50)

Values of $B(\alpha)$ for various values of $\alpha$ are:

$$\alpha = -0.9 \quad -0.5 \quad 0 \quad 1 \quad 2$$

$$B(\alpha) = 0.1310 \quad 0.1503 \quad 1/6 \quad 0.1878 \quad 1/5$$

Hence, from (41), using (48) and (49), $i^*$ is obtained thus:

$$i^* = \frac{mcw^2}{H} \left[ \frac{H_e}{H} \right]^{1/2} \left[ n^*(r_e) \left\{1 - 6B(\alpha)\right\} ight]$$

$$- \frac{dn^*(r_e)}{dr_\ast} \frac{2B(\alpha)}{h_3}$$  \hspace{1cm} (51)

By means of equation 3, 6, 7, and 12 this may be transformed to

$$i^* = \frac{mcw^2}{H_0a^3} \left[ n^*(r_e)r_\ast^{2} D_1(\phi, \alpha) ight]$$

$$- \frac{dn^*(r_e)}{dr_\ast} \frac{r_\ast^3 F_1(\phi, \alpha)}{h_3}$$  \hspace{1cm} (52)

which is valid for the particular pitch-angle distribution (47). Here

$$D_1(\phi, \alpha) = \frac{3\{1 - 6B(\alpha)\}(\cos \phi)^{3+3\alpha}}{(1 + 3 \sin^2 \phi)^{2+1/4 \alpha}}$$  \hspace{1cm} (53)

and

$$F_1(\phi, \alpha) = \frac{2B(\alpha)(\cos \phi)^{3\alpha+3}}{(1 + 3 \sin^2 \phi)^{1/4 \alpha}}$$  \hspace{1cm} (54)

The complete value of $i$ at any point $P$ is given by

$$i = \sum \int i^* dw$$  \hspace{1cm} (55)

integrated over all values of $w$ and summed over all kinds of particle.

4.11. The case $\alpha = 0$; uniform distribution of velocity direction. When $\alpha = 0$, the pitch-angle distribution (47) corresponds to a uniform distribution of velocity direction. This is because $F(w, \theta)$ is then proportional to the zonal area of a sphere of radius $w$ (in the velocity space) lying between the polar angles $\theta$ and $\theta + d\theta$. Moreover, in this case, by (46), $n^*$ has the same value all along any line of force, and by (52) the first term in $i^*$ is everywhere zero. Thus (52) reduces to

$$i^* = \frac{mcw^2}{H_0a^3} \frac{dn^*(r_e)}{dr_\ast} r_\ast^3 F_1(\phi, \alpha) \ (\alpha = 1)$$  \hspace{1cm} (56)

This current is part of the current equivalent to the diamagnetism.

4.12. The case $\alpha \neq 0$. When $\alpha \neq 0$, the velocity distribution is nonuniform. The density of velocity points on the sphere of radius $w$ is proportional to $F(w, \theta)/\sin \theta$. Figure 2 illus-
trates this density as a function of $\theta$ for a few values of $\alpha$. The curves are normalized to give the same mean density in each case. For $\alpha > 0$ the larger pitch angles are in excess; for $\alpha < 0$, the smaller pitch angles.

As $H$ increases away from the equator along any line of force, (46) signifies that if $\alpha > 0$ the number density $n^*$ decreases poleward, whereas for $\alpha < 0$ it increases poleward.

For $\alpha > 0$ the first part of $i^*$ in equation 52 is negative, corresponding to an eastward flow; for $\alpha < 0$ this part is positive, therefore westward.

For all values of $\alpha$, the sign of the second part of $i^*$ in equation 52 is negative (eastward) when $n^*(r_e)$ is increasing outward, and positive (westward) where $n^*(r_e)$ is decreasing outward. If $n^*(r_e)$ increases outward to a single maximum at $r_e = r_{e0}$ and then declines to zero as $r_e$ further decreases, the second part of $i^*$ is eastward for $r_e < r_{e0}$ and westward for $r_e > r_{e0}$. The distribution of $n^*(r_e)$ and of this part of $i^*$ may be more complicated.

4.2. A Special Number-Density Distribution

Our discussion is now further specialized by taking the function $n^*(r_e)$ to have the form

$$n^*(r_e) = n^*e^{-z^2}$$ (57)

where $\eta$ is a constant, and

$$z = (r_e - r_{e0})/a = f_e - f_{e0}$$ (58)

Thus $f_e$ denotes $r_e$ in terms of the earth's radius $a$ as unit, and $f_{e0}$ is the value of $f_e$ at which $n^*$ has its maximum in the equatorial plane; it is the value of $f_e$ for the center line of the belt.

The greater $\eta$, the more rapidly does $n^*$ decrease inward and outward from the center line of the belt. The values of $\eta$ are here given, for which at $z = \pm 1$ the ratio $n^*/n_o^*$ has some particular values:

$$n^*/n_o^* = 0.1 \quad 0.01 \quad 0.001$$
$$\eta = 1.517 \quad 2.145 \quad 2.625$$

Such a Gaussian distribution of number density with $\eta = 1.52$ fits the middle part of $V_2$, the outer Van Allen belt [Van Allen and Frank, 1959], that is, from $f_e = 3.5$ to $f_e = 4.5$.

4.2.1. The current-intensity distribution. The special density distribution given by (46) and (57) leads to the following distribution of current intensity, using (52):

$$i^* = i_0^* \left[ C_i(g, f_o, z) D_i(\phi, \alpha) + E_i(g, f_o, z) F_i(\phi, \alpha) \right]$$ (59)

where

$$i_0^* = n_o^* mcw^2/H_o a$$ (60)

$$C_i(g, f_o, z) = (f_o + z)^2 e^{-z^2}$$ (61)

$$E_i(g, f_o, z) = 2\eta^2 z(f_o + z)^3 e^{-z^2}$$ (62)

For illustration the two parts of $i^*$ have been calculated for many points in the meridian plane, for a hypothetical $V_2$ belt (section 7.3), corresponding to the following values of $f_o, \alpha$, and $g$:

$$f_o = 6 \quad \alpha = -\frac{1}{2} \quad g = 1.517$$ (63)

As shown above, this value of $g$ corresponds to a reduction of the density by a factor 10 at the distances $\pm a$ from the center line of the belt. The calculated values of $i^*$ have been used in drawing the current contour lines, or lines of constant current intensity, shown in Figure 3; (a) and (b) relate to the first and second parts of $i^*$ in (59), and (c) relates to $i^*$ itself. The contour lines are numbered in terms of the value $i^*$ at the center line of the belt, $r_e = r_{e0}$ (or $z = 0$) and $\phi = 0$;

$$i^* = 3f_o^2 \left[ 1 - 6B(\alpha) \right] i_0^*$$
$$= 1.56 \times 10^5 n_o^* m e^2$$
$$= 1.61 \times 10^{-1} n_o^* E^*$$ amperes

where $E^*$ denotes the kinetic energy of the particle expressed in kev.

As was stated in section 4.12, in the present case ($\alpha < 0$) the first part of $i^*$ (Fig. 3a) is everywhere westward. The second part is eastward on the inner side of the belt and westward on the outer side (Fig. 3b). The resultant current $i^*$ is more westward than eastward, but it is eastward, in the equatorial plane, within a radius slightly less than $r_{e0}$. The maximum westward intensity in this plane slightly exceeds 12$i^*$, at about 6.5$a$; the maximum eastward intensity is rather more than 6$i^*$, at about 5.6$a$.

4.3. The Total Ring Current

The total ring current $J^*$ corresponding to $i^*$ is given by

$$J^* = \int \int i^* dS_3 = \int \int i*r_e \cos^4 \phi \, dr_3, d\phi$$
Fig. 3. The proportionate intensity distribution in the model belt $V_s$. The center line of the belt is taken to be at 6 earth radii ($f_0 = 6$), and the pitch-angle distribution is anisotropic ($\alpha = -0.5$). The number density along the equatorial radius decreases from the maximum value $n_s^*$ to $1/10$ in a distance of 1 earth radius towards the inner and outer sides of the belt ($g = 1.517$). (a) The first term in the bracket of (59). (b) The second term in the same bracket. (c) The combined distribution. The combined current intensity at the center line of the $V_s$ belt is taken to be unity.
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using (5). Using (52), this equation may be rewritten

\[ J^* = \left( \frac{mcw^2}{H_0 a^3} \right) \int \left\{ n^* (r_e) r_e^3 D_\phi (\phi, \alpha) \right\} \text{dr}_e \]

\[ - \left( \frac{dn^* (r_e)}{dr_e} \right) r_e^4 F_\phi (\phi, \alpha) \left( \cos^4 \phi \right) \text{dr}_e \text{ d} \phi \]  

In this equation the region of integration is limited by the dense lower atmosphere. We take the meridian cross section of the volume occupied by the particles to be bounded by a closed curve, symmetrical with respect to the equatorial radius, as shown in Figure 4. Let \( \pm \phi (r_e) \) denote the latitudes of the points \( A \) and \( A' \) where the line of force \( r_e \) meets the curve, and let \( r_1 \) and \( r_2 \) denote the values of \( r_e \) where the curve crosses the equatorial plane. Then the limits of integration with respect to \( \phi \) must be \( \pm \phi (r_e) \), and those with respect to \( r_e \) must be \( r_1 \) and \( r_2 \). In place of (64) we obtain

\[ J^* = \left( \frac{2mcw^2}{H_0 a^3} \right) \int \left\{ n^* (r_e) r_e^3 \right\} I_D (r_e, \alpha) \text{dr}_e \]

\[ - \left( \frac{dn^* (r_e)}{dr_e} \right) r_e^4 I_F (r_e, \alpha) \text{dr}_e \]  

where

\[ I_D (r_e, \alpha) = I_D \{ \phi (r_e), \alpha \} \]

\[ I_D (\phi, \alpha) = \int_0^\phi D_\phi (\phi, \alpha) (\cos^4 \phi) \text{d} \phi \]

\[ I_F (r_e, \alpha) = I_F \{ \phi (r_e), \alpha \} \]

\[ I_F (\phi, \alpha) = \int_0^\phi F_\phi (\phi, \alpha) (\cos^4 \phi) \text{d} \phi \]  

Figure 5 shows \( I_D (\phi, \alpha) \) and \( I_F (\phi, \alpha) \) as functions of \( \phi \) for some values of \( \alpha \). As \( \phi \) increases from 0 to \( \frac{\pi}{2} \), \( I_D (\phi, \alpha) \) and \( I_F (\phi, \alpha) \) increases from 0 to maximum values. For each value of \( \alpha \), the variations of \( I_D \) and \( I_F \) are small beyond certain values of \( \phi \). Table 2 gives the maximum values of \( I_D \) and \( I_F \) for several values of \( \alpha \), and also the latitudes \( \phi' \), \( \phi'' \) at which they attain approximately 99 and 90 per cent of their maximum values. The values of \( I_D \) and \( I_F \) and of analogous functions that occur later in this paper have been found by numerical integration.

In connection with the \( V_1 \) radiation belt we consider values of \( r_e \), the lower limit of \( r_e \) in (65), not less than 2.5a. The lines of force \( r_e \geq 2a \) meet the earth's surface at \( \phi \geq 45^\circ \). For the values of \( \phi \) likely to be of physical interest in this problem we may without significant error substitute \( I_D (\pi/2, \alpha) \) for \( I_D (r_e, \alpha) \) and \( I_F (\pi/2, \alpha) \) for \( I_F (r_e, \alpha) \). Thus (65) becomes

\[ J^* = \left( \frac{2mcw^2}{H_0 a^3} \right) \int \left\{ I_D \left( \frac{\pi}{2}, \alpha \right) \int n^* (r_e) r_e^3 \text{dr}_e \right\} \]

\[ - \left( \frac{dn^* (r_e)}{dr_e} \right) r_e^4 I_F \left( \frac{\pi}{2}, \alpha \right) \text{dr}_e \]  

4.31. The total current for the model belt (57). The expression (65) for the total current takes the following form for our model belt for which the density distribution is given by (46) and (57):

\[ J^* = \left( \frac{2mcw^2 n_e a}{H_0} \right) \left\{ I_D \left( \frac{\pi}{2}, \alpha \right) \int C_J (g, f_0, z) \text{dz} \right\} \]

\[ + I_F \left( \frac{\pi}{2}, \alpha \right) \int E_J (g, f_0, z) \text{dz} \]  

Here

\[ C_J (g, f_0, z) = (f_0 + z) C_1 (g, f_0, z) \]

\[ = (f_0 + z)^3 e^{-sz} \]  

\[ E_J (g, f_0, z) = (f_0 + z) E_1 (g, f_0, z) \]

\[ = 2g^2 z (f_0 + z)^4 e^{-sz} \]  

As in the collision integrals of the kinetic theory of gases, the limits of integration with respect to \( z \) may be taken as \(-\infty, +\infty\), probably with an accuracy of 99.9 per cent for the values of \( g \) considered in section 4. Then (69) becomes

\[ J^* = \left( \frac{2mcw^2 n_e a}{H_0} \right) \int \left\{ I_D \left( \frac{\pi}{2}, \alpha \right) \int C_J (g, f_0, z) \text{dz} \right\} \]

\[ + I_F \left( \frac{\pi}{2}, \alpha \right) \int E_J (g, f_0, z) \text{dz} \]  

Fig. 4. The geometry for the integrations \( I_D (\phi, \alpha) \) and \( I_F (\phi, \alpha) \). The curve \( Ar_A \tau_1 \) is the meridian cross section of the belt.
Fig. 5. The integrals $I_D(\phi, \alpha)$ and $I_F(\phi, \alpha)$ as functions of $\phi$ for various values of $\alpha$. For each value of $\alpha$, the variation of $I_D$ and $I_F$ is small beyond a certain value of $\phi$.

$$J^* = 88.20n_0^*E^*(f_0^3/g + 3f_0^2g^2)$$

$$(7.2)$$

This current is in ampere units if $E^*$, the kinetic energy of the particles considered, is measured in kev. Values of the first bracketed factor in (7.2) are given in Table 3 for various values of $f_0$ and $g$.

Values of $I_D(\pi/2, \alpha) + 4I_F(\pi/2, \alpha)$ are here given for several values of $\alpha$:

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$I_D + 4I_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.9</td>
<td>0.8724</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.7338</td>
</tr>
<tr>
<td>0</td>
<td>0.6092</td>
</tr>
<tr>
<td>1</td>
<td>0.4504</td>
</tr>
<tr>
<td>2</td>
<td>0.3576</td>
</tr>
</tbody>
</table>

For our model belt (63), we find that

$$J^* = 9.38 \times 10^3n_0^*E^* \text{ amperes} \quad (73)$$

For the outer radiation belt ($V$) we may adopt

$$f_0 = 3.5 \quad g = 1.517 \quad \alpha = 1 \quad (74)$$

giving

$$J^* = 1.18 \times 10^3n_0^*E^* \text{ amperes} \quad (75)$$

This is less than the value given by (73), mainly because of the smaller cross section. In section 5.11 we discuss the energy spectrum of the particles in the $V$ belt.

4.4 The Dipole Moment of the Ring Current of the Model Belt

At a sufficient distance outside the belt its magnetic field tends to equality with that of a dipole at the earth's center $O$. Let $M$ denote the moment of this belt dipole, and $M^*$ the part of $M$ associated with the part $t^*$ of the current. Then

$$M^* = \pi a^2 \int \int i^*f_4^2 \cos^2 \phi \, dS$$
For the type of belt corresponding to equations 46 and 57, integration on the same lines as in section 4.31 gives the result

\[ M^* = 2.89 \times 10^{18} n_0 E^* \left( \frac{f_0^2}{g} + \frac{5f_0^3}{g^3} \right) + \left( \frac{15f_0}{4g^5} \right) [I_D + 6I_F] \text{ gauss cm}^3 \]

where

\[ I_D(\phi, \alpha) = \int D_n(\phi, \alpha)(\cos^2 \phi) \, d\phi \]
\[ I_F(\phi, \alpha) = \int F_n(\phi, \alpha)(\cos^2 \phi) \, d\phi \]

5. **The Magnetic Field of the Ring Current**

The ring-current intensity is the sum of contributions \( i^* \) corresponding to particles of energy \( E^* \). The corresponding contribution to the ring-current field will be denoted by its components \( H_r^* \) and \( H_z^* \), respectively perpendicular and parallel to the dipole axis. They refer to a point \( P \), specified by its polar coordinates \( r, \phi \) (or \( \theta, \phi \)), or alternatively by \( r_1, \phi \) (or \( \theta_1, \phi \), where \( f = f_0 \cos^2 \phi \), by equation 1).

The ring current can be divided into elements \( i^* d S \) (see equation 5) specified by polar coordinates \( r', \phi' \) (or \( \theta', \phi' \)) or alternatively by \( r_1', \phi' \) (or \( \theta_1', \phi' \), where \( f' = f_0' \cos^2 \phi' \)). Hence [cf. Stratton, 1941, p. 263],

\[ H_r^* = -\left( \frac{2}{ac} \right) \int \left( K(k) - E(k) \right) \cdot \left( \int f_0 \sin \phi - f' \sin \phi' \right) \cdot (E/F) \frac{dS}{F} \]

\[ H_z^* = -\left( \frac{2}{ac} \right) \int \left( K(k) - E(k) \right) \cdot \left( \int f_0 \cos \phi \cdot \cos \phi' \right) \cdot (E/F) \frac{dS}{F} \]

Here \( c \) denotes the speed of light, \( K(k) \) and \( E(k) \) denote the complete elliptic integrals of the first and second kinds for

\[ k^2 = \left( \frac{4f_0 f' \cos \phi \cos \phi'}{F^2} \right) \]

and

\[ F = f^2 + f'^2 + 2ff' \cos (\phi - \phi') \]
\[ F_\perp = f^2 + f'^2 - 2ff' \cos (\phi - \phi') \]

The complete field components \( H_r \) and \( H_z \) are needed if satellite magnetic observations are to be accurately compared with calculations for a model radiation belt. Here we present our calculated results for points in the equatorial plane (\( \phi = 0 \)) only; but we hope later to evaluate the model field more generally.

In the equatorial plane (\( \phi = 0 \)), \( H_r = 0 \), and

\[ H_z^* = -\left( \frac{2}{ac} \right) \int \left( K(k) - E(k) \right) \cdot \left( \int f_0 \sin \phi - f' \sin \phi' \right) \cdot (E/F) \frac{dS}{F} \]

At the earth’s center \( O \), where \( f = 0 \) and thus \( k = 0, E = K = \pi/2, F = F_\perp = f' \),

\[ H_z^* = -\left( \frac{2\pi}{c} \right) \int \left( i^*/f \right) \cdot (E/F) \frac{dS}{F} \]

Proceeding as in section 4.3 as regards the integration with respect to \( \phi' \), we have

\[ H_z^* = \left( \frac{2\mu_0 w^2}{H_0 a^3} \right) \int I_D(\pi/2, \alpha) \int I_F(\pi/2, \alpha) \frac{dS}{F} \]

\[ = \int I_D(\pi/2, \alpha) \int I_F(\pi/2, \alpha) \left( \int n_*(f) \frac{dS}{F} \right) \]

where \( n_*(f) \) denotes the density in the equatorial plane at the radial distance \( af \).

5.1. **The Ring-Current Field at the Earth’s Center for Our Model Radiation Belts**

For the model radiation belt specified by equations 46 and 57, the field \( H_z^* \) at the earth’s center is obtained by substitution from (57) into (80). As in evaluating (69), the limits for the integration with respect to \( z \) can be taken as \( \pm \infty \). The result is

\[ H_z^* = C(\frac{f_0^2}{g} + \frac{1}{2g^3}) \]

\[ \cdot \left[ I_D(\pi/2, \alpha) + 3I_F(\pi/2, \alpha) \right] \]

(81)
TABLE 2. Maximum Values of $I_D$ and $I_F$ and the Latitudes $\phi', \phi''$ at Which They Attain Approximately 99 and 90 Per Cent of Their Maximum Values: for Several Values of $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$-0.9$</th>
<th>0.5</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_D$</td>
<td>0.2256</td>
<td>0.0966</td>
<td>0</td>
<td>-0.1028</td>
<td>-0.1472</td>
</tr>
<tr>
<td>99%: $\phi' = 45^\circ$</td>
<td>42^\circ</td>
<td>34^\circ</td>
<td>31^\circ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>90%: $\phi'' = 28^\circ$</td>
<td>26^\circ</td>
<td>20^\circ</td>
<td>19^\circ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_F$</td>
<td>0.1617</td>
<td>0.1593</td>
<td>0.1523</td>
<td>0.1383</td>
<td>0.1262</td>
</tr>
<tr>
<td>99%: $\phi' = 61^\circ$</td>
<td>56^\circ</td>
<td>50^\circ</td>
<td>42^\circ</td>
<td>37^\circ</td>
<td></td>
</tr>
<tr>
<td>90%: $\phi'' = 42^\circ$</td>
<td>37^\circ</td>
<td>34^\circ</td>
<td>28^\circ</td>
<td>26^\circ</td>
<td></td>
</tr>
</tbody>
</table>

where

$$C = -4\pi^{3/2} m w^2 n_0^* / H_0 = -2.23 \times 10^{-2} n_0^* E^*$$  \hspace{1cm} (82)$$

Values of the first factor in parentheses in (81) are given in Table 4 for various values of $f_0$ and $g$. Values of the last factor in (81) are here given for several values of $\alpha$:

$$\alpha = -0.9 \quad -0.5 \quad 0 \quad 1 \quad 2$$

$I_D + 3I_F = 0.7107 \quad 0.5745 \quad 0.4569 \quad 0.3121 \quad 0.2314$

Hence, for the particular model belt ($V_3$) specified by equation 63,

$$H_{se,*} = -0.306n_0^* E^* \gamma$$  \hspace{1cm} (83)$$

The corresponding magnetic moment of this ring current (section 4.4) is

$$M^* = 1.17 \times 10^{22} n_0^* E^* \text{ gauss cm}^3$$

5.11. The magnetic field $H_{se}$ for the outer radiation belt $V_3$, for an assumed energy spectrum. For the model belt adopted as approximately corresponding to the outer radiation belt ($V_3$), we adopt the numerical data of (74). Then equation 81 and section 4.4 give

$$H_{se,*} = -0.0572n_0^* E^* \gamma$$  \hspace{1cm} (84)$$

and

$$M^* = 0.688 \times 10^{22} n_0^* E^* \text{ gauss cm}^3$$

To obtain the complete value of $H_{se}$ an energy spectrum must be assumed. Van Allen [1959] summarized the information then available as shown in Table 5.

The energy spectrum is still undetermined. Purely for illustration we assume a Maxwellian distribution expressed by

$$n_0^* = \frac{n_0^*}{(2\pi)^{1/2}} \frac{E_m^{-3/2}}{e^{E_m^*/kT} - 1} dE^*$$  \hspace{1cm} (85)$$

where $E_m$ denotes the mean energy. Then

$$\int n_0^* E^* \ dE^* = 3n_0^* E_m$$  \hspace{1cm} (86)$$

Thus, from equations 84 and 86,

$$H_{se} = -0.172n_0^* E_m \gamma$$  \hspace{1cm} (87)$$

The corresponding magnetic moment of the belt (section 4.4) is given by

$$M = 2.07 \times 10^{22} n_0^* E_m$$

The values of $n_0$ and $E_m$ for the $V_3$ belt are not known, either for electrons or for protons. Nor is it known whether $E_m$ is above or below the instrumental cutoff. For electrons we tentatively adopt the values $n_0 = 20/cc$ and $E_m = 10$ kev (just below the instrumental cutoff). Then the magnetic field produced at the earth's center by the electrons is given by

$$H_{se} = -34 \gamma$$  \hspace{1cm} (88)$$

TABLE 3. Values of $f_0^3/g + 3f_0/2g^2$ for Various Values of $f_0$ and $g$

<table>
<thead>
<tr>
<th>$g$</th>
<th>$f_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.5</td>
</tr>
<tr>
<td>1.517</td>
<td>29.77</td>
</tr>
<tr>
<td>2.145</td>
<td>20.52</td>
</tr>
<tr>
<td>2.625</td>
<td>16.63</td>
</tr>
</tbody>
</table>
Table 4. Values of \((f_0^2/g) + (1/2g^2)\) for Various Values of \(f\) and \(g\)

<table>
<thead>
<tr>
<th>(g)</th>
<th>(f_0)</th>
<th>3.5</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.517</td>
<td>8.218</td>
<td>10.69</td>
<td>16.62</td>
<td>23.87</td>
<td>32.44</td>
<td>42.33</td>
<td>53.53</td>
<td>66.06</td>
<td></td>
</tr>
<tr>
<td>2.145</td>
<td>5.762</td>
<td>7.510</td>
<td>11.71</td>
<td>16.83</td>
<td>22.89</td>
<td>29.89</td>
<td>37.81</td>
<td>46.67</td>
<td></td>
</tr>
</tbody>
</table>

Equation 88 suggests a reduction of the horizontal magnetic intensity at the earth's surface of the order of 1/1000 of the normal intensity \(H_0\). This inference is based on the following (uncertain) model of the \(V_1\) belt: (a) the pitch angle distribution is given by (47) with \(\alpha = 1\); (b) the density distribution is Gaussian, given by (57) with \(g = 1.517\); (c) the electron energy spectrum is Maxwellian, with \(n_e = 20/\text{cc}\) and \(E_m = 10\) kev; and (d) the proton contribution to \(H_{sc}\) is not taken into account.

If the \(V_1\) belt completely vanished, equation 88 would imply an increase in \(H_0\) of about 30 gammas. If the value here adopted for \(n_0E_m\) is too high, the value in equation 88 must be reduced, and the disappearance of the \(V_1\) belt would affect \(H_0\) less.

5.12. The ring-current field at the earth's center in terms of the total number of particles. Let \(N\) denote the total number of particles in the belt and \(N^*\) the total of those with energy \(E^*\). They are integrals of \(n(r, \phi)\) or \(n^*(r, \phi)\) throughout the volume occupied by the belt. For a distribution of type (46) or (57), using (8), and adopting limits of integration as before, we have

\[
N^* = 2\pi/B(\alpha) I_\rho \left(\frac{\pi}{2}, \alpha\right) \int n^*(r) r^2 \, dr
\]

\[
= 2\pi^{3/2} a^3 n_0^* (f_0^2/g)
+ 1/2g^3)I_\rho (\pi/2, \alpha)/B(\alpha)
\]

(89)

The factors in (89) have already been given for several values of \(f_0\), \(g\), and \(\alpha\). Hence, for our model \(V_1\) belt (63),

\[
N^* = 7.31 \times 10^{28} n_0^*
\]

(90)

and for the model \(V_1\) belt (74),

\[
N^* = 1.75 \times 10^{28} n_0^*
\]

(91)

Combining (81) and (89), we have

\[
H_{sc}^* = -7.72 \times 10^{-20} B(\alpha)
\cdot \{I_D(\pi/2, \alpha)/I_\rho (\pi/2, \alpha) + 3\} N^* E^* \gamma
\]

(92)

Thus, for the model \(V_1\) belt,

\[
H_{sc}^* = -4.18 \times 10^{-20} N^* E^* \gamma
\]

(93)

and for the model \(V_1\) belt,

\[
H_{sc}^* = -3.27 \times 10^{-20} N^* E^* \gamma
\]

(94)

5.2. The Ring-Current Field in the Equatorial Plane

For the model \(V_1\) belt (63), \(H_{sc}^*\) has been evaluated numerically, using equation 78, at the Computing Center of the University of Michigan, with an IBM 704 Computer.

The integration was replaced by a summation, for 924 (=28 \times 33) elements \(dS_0\) of the cross section of the belt; \(dS_0 = a^3 f_0^* \cos \phi' df_0^* d\phi'\), by (5). All the elements \(df_0'\) had the same value, 0.1, and all the elements \(d\phi'\) had the value \(2^\circ\) or 0.0349 radian. The integrand was taken to be constant over each element \(dS_0\) with the value for the mean \(f_0^*\) and \(\phi'\) for the element. These mean values were taken as follows (note that \(f_0 = 6\) for the model belt considered):

\[
f_0^* = 6 + 0.1 m
\]

\(-12 \leq m \leq 15\)

\[
\phi' = (2n + 1)^\circ
\]

\(0 \leq n \leq 32\)

The value of the integrand outside this region of integration of (78) is too small to need consideration.

The result is to determine the function \(G(f_0,\)
$g, \alpha, f$ in the equation (for the equatorial plane):

$$H_z^* = G(f_0, g, \alpha, f) n_o^* E^* \quad (95)$$

The function $G$ was determined for the values of $f_0, g,$ and $\alpha$ given by (63), over the range $f = 1$ to $f = 10$.

5.21. Comparison with the Explorer VI magnetic measurements. Smith, Coleman, Judge, and Sonett [1960] have published a preliminary account and discussion of the field measurements made by the magnetometer carried on the satellite Explorer VI. Like the measurements made earlier [Pushkov and Dolginov, 1959], on the Russian satellite Mechta, they revealed a notable field perturbation, indicating the presence of an electric current in the space round the earth. As is natural at this stage, they discussed their data in connection with a model current flowing uniformly in a toroid of circular cross section.

Our calculations of the field of a model radiation belt cannot be compared in detail with their data: (a) because our results as yet relate only to the field in the equatorial plane, which was not that of the satellite orbit; and (b) because they measured not the total intensity but its component $H_n$ normal to the spin axis of the payload. When our calculations for our model are more complete, they may be useful for further comparison with satellite data by those who have the full orbital and other accessory information.

The best comparison open to us seems to be with their Figure 2, which refers to August 9, 1959, and is reproduced here as Figure 6(a). It gives on a logarithmic scale their measured values of $H_n$ at orbital points from about 32,000 to 48,000 km from the earth’s center (that is, from about $5a$ to $7.5a$). Using their orbital and spin axis data they were able to calculate what $H_n$ along the orbit should have been were there no disturbance of the earth’s field. They plotted these values, and also those of the total undisturbed intensity. The latter curve differs only moderately from the curve for the undisturbed $H_n$. But the curve of the $H_n$ actually measured shows a large reduction, down to a minimum value of about 15 gammas at a radial distance of $6.2a$ or $6.3a$. The corresponding undisturbed values of $H_n$ and $H$ are about 85 and 130 gammas. They found it possible to fit their $H_n$ results closely to the computed $H_n$ values for a toroidal current as follows:

- Total current, 5 million amperes
- Radius $f_0$ of the center line, 60,000 km ($9.4a$)
- Radius of cross section, $3a$ or less

They remark that there was no penetration of the current by Explorer VI on the said date.

Without reference to the Explorer data, we had adopted the mean radius $6a$ for our model radiation belt $V_s$. The corresponding distribution of current intensity differs markedly from that of uniform flow in a toroid of circular or elliptic cross section, such as was considered by Chapman.
and Ferraro [1933]. Figure 6(c) shows the distribution of current intensity $i^*$ along an equatorial radius, for our model belt (63). It shows also the distribution of number density and of field intensity $H^*$. All three quantities are plotted on linear scales for the following adopted value of the factor $n_0^*E^*$ in our formulas for $H^*$ and $i^*$:

$$n_0^*E^* = 150$$

For example, $n_0^*$ may be 1 particle (proton) per cubic centimeter, and $E^*$ may be 150 kev. The density curve is plotted on this basis. The graph of $H^*$ shows a minimum value of about $-125$ gammas at a radial distance just short of $6a$, and a maximum value of about $+45$ gammas at approximately $7.5a$. The corresponding field at the earth's center $O$ is about 45 gammas.

Figure 6(b) shows the distribution of the combined field, of the earth and the ring current, along the equatorial radius. Its scales are the same as those of Figure 6(a). The graphs in these two figures are not strictly comparable, for the reasons already indicated, but their similarity is notable. This suggests that the Explorer VI data might be interpreted as indicating the presence of a radiation belt somewhat resembling our model, in the region traversed by the satellite. Its center line might be slightly greater than that of our model, perhaps $6.2a$. Note that the combined field has its minimum at a rather greater distance than that of minimum $H^*$. The corresponding net current flow in our model $V_8$ belt is approximately 2 million amperes (cf. equation 73), and consequently notably less than that of the toroidal current considered by Smith, Coleman, Judge, and Sonett [1960]. But it must be remarked that our calculations for our model belt assumed no disturbance of the dipole field. As the observed disturbance in the space occupied by the belt is found to be considerable, as shown also by our calculations for the adopted value of $n_0^*E^*$, further work on this problem is necessary to obtain a self-consistent system of current and field.

The degree of accuracy of the numerical computation of $G$ can be tested by comparison with (83), which gives a more exact calculation of $H^*$, obtained by integration, for the earth's center. The computed value of $G$ in this region is about 5 per cent too large. We think this may be due to the neglect of the currents on the fringes of the $V_8$ belt within $4.8a$ and beyond $7.5a$. The current intensity at the inner limit is rather more than at the outer limit. As the field at the center of a circular line current $i$ of radius $a$ is $2\pi i/a$, it is clear that the neglect of equal and opposite currents beyond the adopted limits might explain the slightly excessive values of $G$ near the earth's center.

5.22. The magnetic moment of the ring current.

The following are the values of certain magnetic moments, including (a) that of the earth, (b) that of the ring current considered by Smith, Coleman, Judge, and Sonett, and those of our model $V_8$ and $V_2$ belts, for the values of $nE$ here adopted.

(a) The earth: $M = 8.1 \times 10^{24}$ gauss cm$^3$
(b) Smith and co-workers: $M = 5.7 \times 10^{24}$
(c) Model $V_8$ belt: $M = 1.8 \times 10^{24}$
(d) Model $V_2$ belt: $M = 4.1 \times 10^{24}$

The smallest of these, for the model $V_2$ belt, is about 5 per cent of the earth's dipole moment; it is less than a quarter of the moment of the $V_8$ belt, which approximates to our suggested interpretation of the Explorer VI data. The toroidal current tentatively considered by Smith co-workers has a moment more than 3 times as large, indeed more than half the moment of the earth. This is so great as perhaps to render their interpretation the less likely of the two; even our $V_8$ ring current has a surprisingly large magnetic moment, enough to affect materially the customary calculations on cosmic-ray impact.

On August 9, 1959, the sum of the $K_3$ 3-hour magnetic indices was 28, indicating magnetic conditions only slightly disturbed. This renders it the more remarkable that the $V_8$ ring current should have added more than 20 per cent to the earth's magnetic moment on that date. During disturbed periods the additional magnetic moment can be expected to be greater than during quiet periods. This is suggested also by the sunspot cycle variation of the annual mean values of $H$ and by the corresponding variation of cosmic-ray intensity.

6. THE MAIN PHASE OF MAGNETIC STORMS

In this section we discuss the main phase of several magnetic storms that occurred during the IGY and IGC 1959. We show how the development of the main phase differs from one storm to another, and we discuss the injection of
solar particles into the region of the ring current. It is interesting to consider first the variation of the monthly mean values of $H$ during the IGY, in the same way as Schmidt [1917], who used the Potsdam $H$ data from 1904 to 1910 [Chapman and Bartels, 1940, p. 293]. A station like Potsdam, in geomagnetic latitude 52.5°N, is much affected by polar magnetic disturbances. Hence it is preferable to use $H$ data for a station in a low geomagnetic latitude, where the effect of polar disturbances is less and the ring-current influence greatest. We have used the data for Koror (geomag. lat. 3.3°S; dip angle $+0^\circ 48^\prime$).

Figure 7 shows the monthly mean values of the horizontal component at Koror observatory during the IGY. The Koror mean value of $H$ obtained from quiet days during the IGY is 37,860 gammas. The monthly mean values (for all days) during the IGY were below this mean except for November 1958. The largest decrease of $H$ occurred in September 1957, and the second largest in July 1958. For September 1957 the deviation from the IGY quiet-day mean was 60 gammas.

Figure 8 shows, for the period from August 25 to October 10, 1957, the changes of the daily mean values of the $H$ component at Koror. Clearly the large September 1957 decrease of monthly mean $H$ in Figure 7 resulted from the appearance of several large but brief magnetic disturbances. Figure 8 shows by arrows the sudden commencements of the nine storms for that month [Lincoln, 1958]. The large impulsive decreases that began on September 2, 12, and 21
resulted from the superposition of two or three successive storms, involving the superposition of successive ring currents. After the minima were reached, the \( H \) component quickly recovered, and regained values approaching or even exceeding the IGY mean value.

The ring current may not die away completely during a magnetically disturbed month. As yet we have no way of knowing the value of the \( H \) component that would correspond to the complete disappearance of the ring current. But Figure 8 suggests that the major part of the ring current that is enhanced during magnetic storms has a rather short life and does not survive more than 10 days if there is no continued injection of solar particles. In the following we discuss the main phases of several magnetic storms that occurred during the IGY and IGC 1959, including the two storms of September 1957 shown in Figure 8.

Our interpretation is based on a physical analysis of \( D \), the disturbing field. We distinguish three main parts of \( D \) [Akasofu and Chapman, 1961], denoting them by the symbols \( DCF \), \( DR \), and \( DP \), which have the following significance: \( D \) for disturbance; \( CF \) for corpuscular flux; \( R \) for ring current; \( P \) for polar. The definitions are as follows:

\( DCF \) field: produced by the electric current flowing near the surface of the hollow carved by the geomagnetic field in the solar stream or cloud that generates the magnetic storm. The current flows as long as the corpuscular flux continues. It is caused by relative motion of the electrons and protons near the hollow surface, as they are turned backward or sideways by the magnetic field. The main effect of the \( DCF \) field at the earth’s surface is to increase \( H \) in low and middle latitudes.

\( DR \) field: produced by enhanced westward electric current during the storm, associated with the motions of energetic particles in the outer geomagnetic field. The main effect of the \( DR \) field during storms at the earth’s surface is to reduce \( H \) in low and middle latitudes. As we state later, we think that a new storm-time belt appears at about 6 earth radii during the storm, well beyond the radius (about 3.5a) at which the normal outer Van Allen belt has its maximum intensity. The \( DCF \) and \( DR \) currents flow at distances of a few earth radii, far above the main terrestrial ionosphere.

\( DP \) field: produced by currents flowing in the ionosphere. They are driven by electromotive forces in the auroral zones. Thence they spread over the main region of the earth between the zones and over the enclosed polar caps. This \( DP \) field has a different time scale from that of the magnetic storm; it may wax and wane more than once during a storm. Each such event, which Birkeland [1908] called a polar elementary storm, is here called a \( DP \) substorm.

Let \( X_{mj} \) and \( Y_{mj} \) denote the instantaneous values of the north-south and the east-west components of the geomagnetic field at station \( j \). The deviation of the above values from a certain base value at time \( t \) reckoned from a certain time \( t = 0 \) may be denoted by \( \Delta X_{mj} \) and \( \Delta Y_{mj} \). The average storm-time variations \( Dst (X_m) \) and \( Dst (Y_m) \) are then defined by

\[
Dst (X_m) = \frac{\sum_{j=-0}^{J} \Delta X_{mj}(t_1, \phi)}{J}
\]

\[
Dst (Y_m) = \frac{\sum_{j=-0}^{J} \Delta Y_{mj}(t_1, \phi)}{J}
\]

where \( J \) denotes the number of stations distributed along a circle of geomagnetic latitude. Here the base value for \( X_m \) and \( Y_m \) may be taken to be the value just before the sudden commencement (ssc) of magnetic storms, and \( t \) may be reckoned from the time of sudden commencement.

The averaging over several observatories eliminates the greater part of the storm field variation that depends on position related to the sun (local time), and it reduces the contribution caused by irregular disturbances \( DP \), mainly proceeding from polar latitudes. If these could be completely eliminated, giving an ideal \( Dst \), then \( Dst (X_m) \) and \( Dst (Y_m) \) would be functions of the storm time \( t \) and latitude \( \phi \) only. In such a case, using the notation given above, they might be written as

\[
Dst (X_m) = DCF(\Delta X_m) + DR(\Delta X_m)
\]

\[
Dst (Y_m) = DCF(\Delta Y_m) + DR(\Delta Y_m)
\]

It must be remembered, however, that each term in the above equations is the combination of the field that originates outside the earth and of a secondary field resulting from the electric current, flowing in the solid earth, that is induced by the variations of the outer field. It is known
Fig. 9. The $\Delta X$ and $\Delta Y$ curves for September 13, 1957, for four zones bounded by circles of geomagnetic latitude (indicated in the figure). The rapid growth and decay of the ring current occurred at 00h 47m GMT on September 13, 1957.
Fig. 10. The Dst (X_m) and Dst (Y_m) curves for September 29, 1957, for four zones bounded by circles of geomagnetic latitude (indicated in the figure). The ssc occurred at 00h 16m GMT on September 29, 1957. Note that the ring current began to grow at about 13h 20m GMT, about 13 hours after the ssc.
that about \( \frac{2}{3} \) of the observed change corresponds to the external field [Chapman and Bartels, 1940, p. 692].

We first consider the storms of September 13, 1957 (Fig. 9), and September 29, 1957 (Fig. 10). In both storms it happened that the ssc's occurred close to Greenwich midnight, namely 00h 47m GMT September 13 and 00h 16m September 29, 1957. There were no significant magnetic disturbances between the Greenwich midnight and the times of the ssc's. Thus we take the base value for \( X_m \) and \( Y_m \) to be the value at Greenwich midnight in each case, and storm time is reckoned from that time. In order to show the latitude dependence of the \( Dst \), we divide the earth's surface into four zones, bounded as follows by circles of geomagnetic latitude:

- **Group I**: Between \( \pm 5^\circ \)
- **Group II**: From \( 5^\circ \) to \( 20^\circ \) and from \( -5^\circ \) to \( -20^\circ \)
- **Group III**: From \( 20^\circ \) to \( 40^\circ \) and from \( -20^\circ \) to \( -40^\circ \)
- **Group IV**: From \( 40^\circ \) to \( 60^\circ \) and from \( -40^\circ \) to \( -60^\circ \)

Figures 9 and 10 show the \( Dst \) (\( X_m \)) and \( Dst \) (\( Y_m \)) curves thus obtained. The contributing observatories in the four zones are listed in the Appendix.

The \( Dst \) (\( X_m \)) curves in Figure 9 show a notable ssc of order 100 gammas at 00h 47 m GMT on September 13, 1957. We ascribe this to the DCF field. The magnetograms from the polar region (which are not given in this paper) show that the ssc was soon followed by active DP substorms. As happens in most (if not all) great magnetic storms, the DCF field was quickly overpowered by the DR field. This occurred about 2 hours after the ssc. The \( Dst \) (\( X_m \)) curve attained its minimum at 09h 00m, about 8 hours after the ssc. We estimate that the total DR field at the earth's surface was at least of the order of 600 gammas. Thus the external part of \( H_m \) was about 400 gammas. The ring current began to decay quickly after 09h, when also the major DP disturbances ceased. Small irregularities superposed on the \( Dst \) (\( X_m \)) curve in Figure 9 are attributed to the incomplete elimination of the effects of DP substorms. \( Dst \) (\( Y_m \)) shows a small increase of the order of less than 50 gammas. Comparing \( Dst \) (\( X_m \)) with \( Dst \) (\( Y_m \)), it is clear that the main DR variation was produced by a westward current.

The \( Dst \) (\( X_m \)) and \( Dst \) (\( Y_m \)) curves in Figure 10 show a less-marked DCF increase at 00h 16m GMT on September 29, 1957. The ring current seemed to become appreciable around 06h but decayed or was overcome by the DCF field at about 12h. A remarkable growth of the DR field began at about 13h 20m. It attained its maximum at 17h. Many large DP substorms are superposed on the \( Dst \) curves. They make it difficult to determine the exact course of the DR change. It seems that the ring current remained strong until 23h, that is, for about 6 hours, and then began to decay.
It is interesting to compare the $D_{st} (X_m)$ curves with the $H$ magnetograms (Fig. 11) from 12 stations along the auroral zone. These show mainly the $DP$ substorm activity. A $DP$ substorm is generally intense only over a part of the auroral zone, so that it may scarcely be indicated by the records of observatories elsewhere. But 12 observatories well distributed along the zone make it possible to record most large $DP$ substorms. On September 29 the auroral zone was rather quiet until 12h except at about 05h, when a small $DP$ disturbance was recorded. Remarkable $DP$ disturbances began at about 13h 30m. Though very transient and intermittent, they remained active until the end of the day. Comparing Figures 10 and 11, it seems that the $DR$ and the $DP$ activities were apparently closely correlated. Both the $DR$ and $DP$ activities were moderate until about 13h 30m, when they suddenly became active. They continued to be active until the end of the day.

Figure 12 shows $D_{st} (H)$ curves for the July 11 and the July 15 storms. They give the average variation of $H$ derived from 12 observatories well
distributed around the earth between latitudes ±45° (see Appendix). The difference between $D_{st} (H)$ and $D_{st} (X, ,,)$ is very small for stations whose latitudes are less than 45°. We have discussed the differences between the two storms in detail in a recent paper [Akasofu and Chapman, 1960]. There was little indication of large DP substorms in the auroral zone during the remarkable initial phase (mainly $DCF$) of the July 11 storm; this phase lasted for at least 7 hours. Although the earth was presumably within an intense solar stream, the small $DR$ and $DP$ activity suggests that only a small part of the solar gas was trapped during that time. Only one appreciable $DP$ substorm occurred (at about 23h 30m GMT on July 11). On the other hand, the initial phase of the July 15 storm was interrupted by an abnormally large $DP$ substorm about 30 minutes after the ssc. Throughout this storm both the $DP$ substorms and the $DR$ field were considerable.

Figure 13(a) shows the $D_{st} (H)$ curve for the August 16-18, 1959, storm; the curve refers to 13 low-latitude observatories (see Appendix). This storm was characterized by prolonged $DR$ activity and many successive small substorms which continued for about 48 hours.

The above several examples indicate that magnetic storms can develop in different ways, particularly as regards the course and intensity of the $DCF$, $DR$, and $DP$ substorm fields. The magnetic records suggest that in some storms a considerable amount of solar gas is trapped, and in others only a little. We think that a simple regular neutral ionized stream, of the type hitherto mainly considered in theories of magnetic storms, can produce the $DCF$ field. The intensity of this field is not necessarily correlated with the intensity of the main phase and the ring current. The development of the main phase and of the $DP$ substorms probably depends on irregularities embedded in the solar streams emitted from time to time from the sun. The degree of capture of solar particles from such irregularities may control the variation of the $DR$ field and of the $DP$ substorms.

We believe there is reason to think that, when the intensity of the ring current is sufficiently increased, the field direction may be reversed in certain limited regions, forming neutral lines in or near the equatorial plane on the inner side of the ring current. Recently we suggested a new theory of the aurora, based on the postulated existence of such neutral lines [Akasofu and...
A diffuse aurora may be produced by high-energy electrons issuing from a narrow strip close to the neutral line, located in the outer part of the outer radiation belt. We showed that the development of the DP substorm coincides with the change of the aurora from the diffuse to the active form. We think that the active aurora is produced by a slight eastward electric field along the neutral line. The eastward electric field may arise from the differential motion of protons and electrons injected from irregularities in the solar stream, as mentioned above. If so, DP substorm activity can be interpreted as indicating injection in or near a region where there is a neutral line. (Neutral lines are of two kinds, X and O; here we refer to an X line on which the magnetic force lines cross.)

As Figure 11 indicates, DP substorms are short-lived phenomena. In general, several large DP storms appear during a medium magnetic storm. The injection may occur when irregularities in the solar stream traverse the space around the earth. From this point of view, the injection of solar particles during the course of the storms discussed above may be summarized as follows:

**September 13 storm.** The ssc was followed for about 10 hours by a number of large intermittent injections. The remarkable ring current began to develop about 2 hours after the ssc. It attained its maximum intensity at 09h 00m (about 8 hours after the ssc). The major injection ceased at about the same time, and the ring current quickly began to decay.

**September 29 storm.** Large injections began about 13 hours after the ssc. A notable ring current started at about the same time. The intermittent injections lasted for about 10 hours, during which time the ring current remained strong. The injection ceased at the end of the day, and then the ring current began to decay.

**July 11 storm.** No appreciable injection occurred for about 7 hours after the ssc, and during that period the earth was enclosed by an intense regular solar stream. Only one appreciable injection occurred, at about 23h 30m, but nothing particular later. The intensity of the ring current was very small.

**July 15 storm.** The ssc was followed by a great number of large injections for about 10 hours or more. The development of the ring current was considerable.

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**August 16–18 storm.** The ssc was followed for about 48 hours by a large number of small injections. The DR activity was not large, but it was prolonged and remained strong for more than 48 hours.

### 7. The Ring-Current Belt

#### 7.1. Depletion of the Outer Radiation Belt during the Main Phase of Magnetic Storms

The several examples of magnetic storms discussed above suggest that many solar particles are trapped and produce the DR field, although there is great variety in the course of the trapping from one storm to another. In 1958, when the radiation belts were found, it was expected that the outer radiation belt would clearly show considerable solar control. It seemed likely that it would be enhanced during magnetic storms so as to become a main seat of the westward ring current whose existence was inferred from the magnetic observations at the earth's surface.

Recent satellite observations have indicated that the observed particles of the outer belt are sometimes markedly depleted during the main phase of storms. Table 6 lists these observations.

An example is given in Figure 13(b), which shows the counting rate at the center of the outer radiation belt (24,000 km or 3.76 radii from the earth's center), during the storm of August 16–18, 1959 [Arnoldy, Hoffman, and Winckler, 1960]. Pass 18, on August 17, during the main phase of the storm, showed a marked reduction of the counting rate, which fell to 2000/sec. Then the rate increased to 18,000/sec during pass 19, nearly at the end of the main phase. The depletion concerned protons of energy 30 Mev or more and electrons of energy 30 kev or more; so far particles of less energy are undetected.

### Table 6. Some Observations of Depletion of the Radiation Belts during Magnetic Storms

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Magnetic Storm Details</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explorer IV</td>
<td>September 3–4, 1959</td>
<td>Rothwell and McIlwain, 1960</td>
</tr>
<tr>
<td>Explorer VI</td>
<td>August 16–18, 1959</td>
<td>Arnoldy, Hoffman, and Winckler, 1960</td>
</tr>
<tr>
<td>Explorer VII</td>
<td>March 31–April 3, 1960</td>
<td>Van Allen and Lin, 1960</td>
</tr>
</tbody>
</table>
Information about the magnetic field measured by the satellite is not available at present. The change in the radiation belt shown in Figure 13(b) is quite different from what the magnetic records made at the earth's surface would suggest. Arnoldy, Hoffman, and Winckler [1960] showed that the counting rate decreased by a factor 1/5 during pass 18, and then increased again by a factor 2. If the model defined by the conditions of section 5.11 represents the V₂ belt reasonably well, and if the number density n₀ changed in proportion to the counting rate, the corresponding \( H_{zc} \) values for electrons would be respectively \(-17\) and \(-68\) gammas (cf. equation 84). If, however, the V₂ data used in (88) are excessive, these changes would be smaller, perhaps much smaller. Thus it seems that the large changes of the V₂ outer radiation belt found so far do not much affect the intensity of the magnetic field at the earth's surface. They are small compared with the observed DR changes during great storms. However, the field changes near and in the V₂ belt may be large enough to be detected by satellites.

7.2. The Depletion of the V₂ Outer Radiation Belt and Auroral Particles

Suppose that in the equatorial plane a cloud of particles is captured from the solar stream. The particles will encircle the earth because of the motions described in section 2, although for a time there may arise a slight electric field tending to prevent their independent motion. The electric field will soon be reduced, and the particles may be expected eventually to spread around the earth, as in the Argus experiment. Suppose also that the pitch-angle distribution is isotropic when the particles are captured. This means that the density of velocity points on the sphere of radius \( ω \) is uniform. From Fig 1, however, we may infer that the particles with pitch angle \( θ_p \) less than 3° have very little chance to go back from their mirror points to the equatorial plane, because they will be absorbed when they pass through the dense atmosphere. Table 1 shows that the time of oscillation \( T_o \) for 30-kev electrons, which are supposed to play an important role in the excitation of auroral luminosity, is of the order of only 2 seconds.

Therefore, whatever the initial condition may be, the particles in the captured cloud that have small pitch angle (and small magnetic moment \( μ \)) will be lost in a few seconds. Then the distribution of velocity points will no longer be uniform. It will have antipodal 'holes' on the surface of the 'velocity sphere,' around the axis \( H \). In a steady state there will then be no electrons with small \( μ \), unless there is a mechanism that continuously produces them.

In a diffuse auroral arc the flux of high-energy electrons is of the order of \( 10^7 \) to \( 10^8/cm^2\ sec \). During an extremely active auroral phase the flux may become as high as \( 10^{11}/cm^2\ sec \). Such electrons must have an extremely small magnetic moment. Furthermore, in order to explain the 'thin-ribbon' structure of the aurora, the width of the source must be extremely narrow, of the order of 30 km or less, if it is in the main normal radiation belt V₂. High-level atmospheric scattering of energetic electrons whose mirror points, if undisturbed, would be well above auroral levels may produce a weak spray of electrons with small \( μ \) outside the visible auroral 'ribbon.' In fact, Meredith, Gottlieb, and Van Allen [1955] observed by rockoons a continuous precipitation of electrons in a wide zone of latitude, including the auroral zone. This may be a 'leakage from the large reservoir,' namely the V₂ belt. But this process cannot produce a curtain-like structure of the aurora.

In a recent paper giving a new theory of auroral morphology [Akasofu and Chapman, 1961], we infer that the ring current may reverse the direction of the earth's field and produce neutral lines when the current is considerably enhanced. The neutral line must lie in or close to the equatorial plane. Because of the rapid gyration and the small cyclotron radius of electrons of high energy (see Table 1), the only place with a width similar to the radius of gyration of such electrons is a narrow strip close to the X-type neutral line. There the guiding-center approximation is not valid and the magnetic moment \( μ \) can change. The electrons that pass through this narrow strip during their oscillatory motions between the mirror points will temporarily lose their magnetic moment \( μ \), because they do not gyrate, and small magnetic irregularities may produce a group of particles with different \( μ \) there.

At a distance of 6 earth radii from the earth's center, the flux of high-energy electrons, according to satellite measurements, is about \( 1/100 \) of that at the center of the V₂ belt: \( (10^{11}/cm^2\ sec) \).
In order to explain the electron flux for a diffuse arc, namely \(10^7 \text{ to } 10^8 \text{ cm}^{-2} \text{ sec}^{-1}\), the process of conversion of the magnetic moment \(\mu\) must be anisotropic, because the fraction of electrons with pitch angle \(\theta\), less than \(3^\circ\), is only of the order of

\[
\left\{2\int_0^{3^\circ} \sin \theta d\theta / \int_0^\pi \sin \theta d\theta \right\}
\]

which has the value \(1.31 \times 10^{-3}\).

An isotropic process can produce a flux of an order no greater than

\[
(10^7 \text{ cm}^{-2} \text{ sec}) \times 1.37 \times 10^{-2}
\]

\[
= 1.37 \times 10^7 \text{ cm}^{-2} \text{ sec}
\]

Note that for an isotropic pitch-angle distribution the number density along a line of force is the same at all points. Even the center of the \(V_s\) belt could not supply the large flux, of the order of \(10^{11} \text{ cm}^{-2} \text{ sec}^{-1}\), required for an active aurora, for more than a few seconds.

We think that for both diffuse and active auroras the necessary reduction of magnetic moments \(\mu\) can only be made in strips close to neutral lines. For active auroras we suppose that both high-energy electrons and the ambient electrons (of much lower initial energy) are accelerated by an eastward electric field along the \(X\)-type neutral line. This may be produced by a slight charge separation of particles while a captured cloud of particles is spreading around the earth, and closing to a ring, on the dark side. A substantial increase of the flux of electrons with energy of the order of 5 kev, as observed by McIlwain [1960], may indicate this acceleration of low-energy ambient electrons. The type B aurora, which is supposed to be produced by high-energy electrons of the order of more than 100 kev during an active phase, may correspond to the acceleration of high-energy electrons in the \(V_s\) belt. It may be noticed that such acceleration by an electrostatic field is most easily possible where the guiding-center approximation loses its validity, as it does near an \(X\)-type neutral line [cf. Akasofu and Chapman, 1961].

For either diffuse or active auroras, if the neutral line stays at one place for more than a few minutes, most of the electrons that pass near it may find their way into the auroral ionosphere. It is interesting to note that even the so-called quiet arcs (we prefer to call them diffuse arcs) continuously change in brightness and always show some motion. Where the neutral line sweeps across the outer belt the number of high-energy electrons will be considerably reduced. This may be the cause of the depletion of the outer belt. In fact, the depletion seems to be greatest in the outer part of the \(V_s\) belt.

7.3. The Storm-Time \(V_s\) Belt

7.31. The auroral zones and the storm-time \(V_s\) belt. Auroral arcs most frequently lie near the circle of geomagnetic latitude \(67^\circ 30'\). In our theory of the aurora, we infer from this that the neutral lines are most often formed at about 6.3 earth radii. If so, the central line of the ring current should lie a little beyond this distance. During a magnetic storm the auroral arcs move to somewhat lower latitudes, and the \(DR\) field increases. The neutral line and the ring current then draw inward, toward the earth. After the \(DR\) field and the ring current begin to decay, the arcs quickly return poleward. Therefore, we may infer that during a magnetic storm there is an additional ring-current belt or storm-time belt \(V_s\), corresponding to the auroral zones.

7.32. The ring-current particles. The observations of the radiation belts by the counters carried by satellites relate as yet to protons of energy higher than about 30 Mev and electrons of energy higher than 30 kev. We have no indication of any enhancement of such high-energy particles during storms. However, as we have seen above, in order to produce the main phase of storms, particles in large numbers must be captured during a storm. We infer that the energy of the particles that mainly produce the ring current during the storm is below the cutoff of the instruments so far carried by satellites. The revolution time \(T_R\) in Table 1 bears on this problem. Suppose that the capture of the particles occurs in a volume rather small compared with the dimensions of the radiation belts, e.g., in a volume comparable with that of the solid earth. The particles will soon encircle the earth (section 7.2). Then, the maximum \(H_{cr}\) should occur when the cloud of particles has completely encircled the earth and the ring is closed. This requires a time \(T_R\). In the September 13, 1957, storm, the whole time from the \(ssc\) to the maximum phase (minimum \(H\)) was about 9 hours. This may be taken as an upper limit for \(T_R\). The times of growth and decay of the
several intermittent DP substorms that occurred during this interval suggest smaller values of $T_R$, of the order of an hour or even less. As yet, we do not know how the pitch angles $\theta_e$ are distributed when the particles are captured. Table 1 shows for $\theta_e = \sin^{-1} 0.1$ the order of magnitude of $T_R$ for electrons and protons of different energies. Clearly 80-kev electrons satisfy the above requirement for $T_R$. If they make the main contribution to the ring current, they must be enhanced during the main phase. This, however, is not the case—rather the contrary (see Fig. 13). Electrons of less energy, 10 kev or below, cannot in this way contribute much to the ring current, because of their large $T_R$.

Therefore, protons with energy below the instrumental cutoff seem to be the particles more likely to produce the storm-time ring current. In Table 1, the time $T_R$ for protons of energy from about 50 kev to 5 Mev meets the requirements for $T_R$ and for the validity (except near neutral lines) of the guiding-center approximation. Equation 81 shows that $H_{zc^*}$ is directly proportional to the energy $E^*$.

$$H_{zc^*} = -0.306 n_e^* E^* (\text{kev}) \gamma$$

Taking the energy to be 500 kev,

$$H_{zc^*} = -153 n_e^* \gamma$$

Thus our model belt, if populated by 500-kev protons whose number density $n_e^*$ is 1/cc, could produce a disturbance $H_{zc^*}$ of the order of $-153$ gammas. In order to produce the same $H_{zc^*}$ the number density $n_0^*$ of 50-kev particles must be 10/cc. These must, of course, be crude estimates, because such a belt of protons would greatly distort the earth's dipole field there. We hope in a later paper to discuss how these values of $n_e^* E^*$ must be modified when the local distortion of the field by the ring current is taken into account in determining its magnetic field at the earth.

From (90) we see that the total number $N$ for 500- and 50-kev protons for $H_{zc^*} = -153$ gammas is respectively of the order of $7.4 \times 10^{28}$ and $7.4 \times 10^{26}$. Suppose that such particles are trapped during a typical DP substorm that lasts 2 hours ($=7.2 \times 10^3$ sec), and that the capture is from a cross-sectional area $\pi a^2$ of the solar stream. This is small compared with the suggested frontal area presented by the $V_s$ belt, of the order of $\pi(6a)^2$. The rate of capture for 500-kev protons during a DP substorm is then

$$7.4 \times 10^{28}/\pi a^2 \times 7.2 \times 10^3 \sec = 8.0 \times 10^4 /\text{cm}^2\sec$$

This may be compared with the flux that produces the $DCF$. Taking the speed to be of the order of $10^8$ cm/sec and the number density to be 10, the $DCF$ flux is $10^5$ cm$^2$/sec. The supposed rather uniform flux of particles responsible for $DCF$ seems likely in large part to be reflected or scattered back at the surface of the solar stream [cf. Chapman and Ferraro, 1940; an article by Chapman and Kendall entitled 'An idealized cylindrical problem of plasma dynamics that bears on geomagnetic storm theory; oblique projection,' is in preparation]. Thus it would not contribute to the capture. We think this occurs when irregularities (in which protons with energy of a few hundred kev may be involved) pass by the earth. Thus, the protons associated with these irregularities may number only 1 in 100 of the whole number density. Such irregularities may be embedded in a fairly uniform stream or cloud, and move with it. Then their velocity is of the order of $10^8$ cm/sec. The duration of a typical DP substorm is about 2 hours. If this indicates the time during which the capture occurs, the length of the irregularities may be of the order of $7200$ sec $\times 10^8$ (cm/sec) = $7.2 \times 10^4$ km. This is about 1/20 of the distance between the sun and the earth. If their cross-sectional radius is of the order of $a$, they will be like threads in the stream or cloud. In a typical storm we observe perhaps 5 or 6 large DP substorms. The development of the ring current may depend on the distribution of such irregularities along the sun-earth line.

8. Concluding Remarks

As was stated in section 5.21, the calculations of this paper, based on an undistorted dipole field, need to be extended to obtain a self-consistent system. The distortion of the field by the $DCF$ currents must also be taken into account as well as the fact that conditions are changing instead of steady as here considered. Further, the multiple structure of auroras indicates, in our view, a more complicated structure of the belt than we have assumed. Thus the
calculations of this paper provide only one step toward an understanding of the complicated phenomena here reviewed.

APPENDIX

Results from the following magnetic observatories are used in this paper.

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<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
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<tbody>
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* Contributed to Figure 12.
† Contributed to Figure 13.

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