The Geomagnetic Field in Space, Ring Currents, and Auroral Isochasms

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Abstract. Using a 48-coefficient spherical harmonic expansion of the geomagnetic field for 1955.0, the results of an analysis due to Finch and Leaton, it is shown that geomagnetic field lines appear to interlink the northern and southern auroral zones. Theoretical average auroral isochasms are also estimated using an integral invariant for particle motion along the lines of force and between the northern and southern hemispheres. Some conditions pertinent to simultaneity and similarity of the aurora borealis and the aurora australis are discussed.

Though the auroral zones northern and southern shift in latitude with time, in accord with the intensity of magnetic disturbances, the average auroral zone remains a useful parameter in polar studies. The geographical distribution of isochasms appears to be successfully calculated from the geomagnetic field alone and an integral invariant of particle motion.

The intersections of particular geomagnetic field lines with the earth's surface in the northern and southern hemispheres, or conjugate points, are indicated.

Introduction. It has long been noted that auroral rays appear aligned more or less closely to geomagnetic field lines in the upper atmosphere [Chapman and Bartels, 1940]. Early theoretical studies of Störmer [1907] proposed that certain auroral particles spiraled down a line of geomagnetic force to a height a few hundred kilometers above the ground where they were reflected at a suitable magnetic field value. In the absence of collisions a particle in the northern hemisphere would then spiral and move along a line of force into the southern hemisphere where in the same way the reflection process would be repeated. This early theoretical work hence raised the question of the possible simultaneity and similarity of auroral displays at opposite intersections of geomagnetic field lines with suitable atmospheric levels of the northern and southern hemispheres.

Other theoretical workers [Chapman and Ferraro, 1931, 1932, 1933; Ferraro, 1952; Alfvén, 1950, 1955; Martyn, 1951; Vestine, 1947] have noted that charged particles may leave a ring current distributed symmetrically about the equatorial plane and penetrate the earth's atmosphere into the auroral zones to produce aurora and polar geomagnetic disturbances. If particles leave a current ring, they should then arrive at auroral levels along a specified auroral zone. The position and shape of the ring current, and the field lines intersecting it, may determine the zone of maximum auroral frequency. Conversely, given an average auroral zone, points upon it constrained to move along geomagnetic field lines should intersect a possible ring-current source. It may be expected that these lines of force ending at the northern average auroral zone when followed southward should provide a conjugate zone in antarctic regions. In a previous paper the authors have compared the southern conjugate zone so derived, with observation, using the first 8 Gauss coefficients of the spherical harmonic representation of the earth's main field [Vestine and Sibley, 1959]. In the present paper this work is further discussed and extended, by means of the first 48 Gauss coefficients, using results of an analysis due to Finch and Leaton [1957]. An examination is made also of the intersection of the estimated field lines connecting the northern and southern auroral zones with the earth's equatorial plane. This study suggests that the supposed ring current, comprising transient drifting charge distributions in outer reaches of the radiation belt from which portions may drain at least weakly on a continuous basis into auroral regions, has an equatorial cross section of oval form.

Finally, the changes in the mirror point of trapped radiation with height above each auroral zone are discussed in relation to auroral
and geomagnetic phenomena. It is concluded that drifts of particles due to space-gradients of the geomagnetic field may also be of significance.

The average auroral zones and isochasms inferred if auroral particles are drained from the outer reaches of the Van Allen radiation belt (at least on a temporary basis) are estimated using integral invariants. These results proved so successful that the Störmer spiraling motion of trapped radiation between northern and southern hemispheres seems likely to be of considerable significance in the case of aurora. Their east-west drift seems to offer an unforced explanation of the averaged directions of auroral arcs. Thus clouds of particles may be substantially trapped for a period of some minutes to several hours within high interzonal levels of the geomagnetic field, perhaps meanwhile being replenished directly from solar or other sources.

**Geomagnetic lines interlinking the auroral zones.** The average auroral zones [Chapman and Bartels, 1940; Vestine, 1944; Vestine and Snyder, 1945] encircle the earth in high latitudes. At the zone of maximum auroral frequency aurora may be observed almost every clear night, and as one departs from this zone, to the north and to the south, in either hemisphere, the average auroral frequency decreases. The authors have shown that the observed northern and southern auroral zones appeared to be interlinked by the geomagnetic field lines. As remarked earlier, these calculated lines were based on the first 8 Gauss coefficients [Vestine and Sibley, 1959]. The lines are recomputed here, using 48 coefficients in the spherical harmonic expansion for the earth's main magnetic field. Figure 1 shows the average northern auroral zone and its computed conjugate auroral zone in the antarctic projected on the earth's equatorial plane. Also shown is the estimated southern auroral zone [Vestine and Snyder, 1945], based on most available auroral observations prior to the International Geophysical Year. It will be noted that the conjugate and 'observed' auroral zones are very nearly coincident in some areas where observations have been more frequent. A region of this kind is the area immediately south of Australia and New Zealand. In very high southern latitudes the discrepancy rises to several degrees of latitude, e.g., near the geographic south pole. In this region the 'observed' auroral zone was based largely upon extrapolated information and the discrepancy is not surprising. Accordingly, it seems likely that the northern and southern isochasms of maximum auroral frequency are

![Fig. 1. Average northern and southern auroral zones as seen from above North Pole. Computations based on 48 Gauss coefficients for 1955.](image-url)
interlinked by geomagnetic field lines. Preliminary examination of the recent IGY observations at Ellsworth Station (lat. = 77°43'S, long. = 318°52'W), kindly sent the authors by Dr. N. Oliver, of the Air Force's Geophysical Research Directorate, indicate results that appear roughly compatible with present estimates based on the northern auroral zones. Results for the South Pole during the IGY indicate that the auroras most frequently observed are in the area between the South Pole and Ellsworth station, though closer to Ellsworth station. Hence the concept of interlinkage of the northern isochasm of maximum auroral frequency in northern Canada (if correctly estimated) with the isochasm in the antarctic appears to predict a southern auroral zone in good accord with observation, though there remains to be discussed a discrepancy of considerable amount between the calculated and probable observed position of the southern auroral zone near the south pole.

Recently Hultqvist [1959] has suggested, on theoretical grounds, that the northern isochasms derived by Fritz [1881] and revised by Vestine [1944] appear to require further revision in the region near James Bay, in central Canada. He concludes that if the isochasms near James Bay are displaced about 4° of latitude farther south, the auroral particles in the ionosphere might arrive there by spiraling down the lines of force from a circular current ring near the equatorial plane. In fact, Gartlein [1959] has proposed a revision of this very amount, or rather has indicated that observations during the sunspot maximum years of the IGY tend to indicate a more southerly position near James Bay. There is, therefore, need for a detailed examination of the IGY and earlier data in this connection, since resolution of the isochasm of maximum auroral frequency by visual observations on the basis of a single year of data is not very precise. It may be useful also to employ magnetic disturbance data, assuming that the average electric currents causing magnetic bays flow along the average auroral zone [Vestine, 1944; Vestine and Snyder, 1945].

This can be done when data of the IGY becomes more conveniently available. Meanwhile, certain interesting aspects, which are perhaps more fundamental than those considered by Hultqvist, may be brought to bear on the problem. Since the data of Figure 1 suggests that the auroral particles may spiral between the northern and southern hemispheres it is interesting to approach this and other related matters in terms of parameters of auroral particle motion.

**Field values at conjugate auroral points.** It is not yet known how auroral particles congregate to produce the various auroral forms. They may be drained from temporary plasma groupings within the radiation belt or from solar streams directly to produce auroral displays in the ionosphere and below [Van Allen, McIlwain, and Ludwig, 1959a]. It has been correctly argued at times that electrons are driven both upward and downward to produce auroral rays. It has also been suggested that certain ions when driven upward cause the illumination of sunlit aurora at heights as great as 1000 km above the earth. There are also experimental results indicating that protons with energy of 50,000 volts or more appear to move downward during the occurrence of homogeneous auroral arcs. It appears

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Fig. 2. Values of the integral invariant I, for unit earth's radius, epoch 1955.0.
certain that when auroral rays occur there are energizing particles spiraling up or down lines of force of the geomagnetic field. Except for those absorbed by direct collisions in the atmosphere there must be many that on moving downward have pitch angles relative to the magnetic field such that they must be reflected for a certain magnetic field strength. After reflection they then must move upward, and many will proceed to auroral regions in the opposite hemisphere. Here they would again be reflected. The condition for reflection is given approximately by the Alfvén equation

$$F/\sin^2 \alpha = \text{const} \quad (1)$$

When reflection occurs $\alpha = 90^\circ$, and $F = F_m$. Consequently

$$F/\sin^2 \alpha = F_m \quad (2)$$

As Alfvén, following Ross Gunn, has shown, particles drift in the presence of a field gradient. The drift velocity $v_d$ is approximately

$$v_d = v_\perp \rho F \times \text{grad } F/2F^2 \quad (3)$$

where $v_d$ is the drift velocity owing to the gradient of field, and $v_\perp$ the particle velocity transverse to $F$, and $\rho$ is the radius of curvature of the line of force [Alfvén, 1939, 1950, 1955, 1958; Singer, 1957]. In the same way additions resulting from centrifugal force and gravity yield a drift current so that a group of auroral rays would spread into a sheet or arc roughly directed as $F \times \text{grad } F$. This appears to be approximately true for the averages of many auroral arcs noted on polar expeditions. In Figure 2, some results for the first and second International Polar Years are indicated for auroral arcs (the curves $I = \text{constant}$ will be discussed later). Inspection of isomagnetic charts of the period indicates a good rough compatibility with alignment of the arcs in the approximate direction $F \times \text{grad } F$. Since the field vectors of

![Fig. 3. Contour lines corresponding to equal integral values $I$, northern hemisphere, geomagnetic coordinates.](image)
disturbance are transverse to the arc directions, it is tempting and certainly simplest to assume that the magnetic disturbance arose from the drift of charged particles, unless this hypothesis must be rejected for other reasons. In fact, since this current can be estimated from geomagnetic data, the downward flux of auroral particles can also be estimated. Thus [Welch and Whitaker, 1959]

\[ v_d \sim v_1 \frac{\rho F \times \text{grad} F}{2F^2} + v_2 \frac{1}{Rw} (v_d/v_2) \]  

where \( \rho \) is the spiral radius of the particle, \( v_1 \) the velocity along the line of force, \( R \) the radius of curvature of the line, and \( w \) is the Larmor frequency. For a dipole equation 4 becomes roughly

\[ v_d \sim \frac{3v_1 \rho}{2r} + \frac{v_2^2}{Rw} \]  

but since \( v_2 \) becomes zero at reflection and \( v_1 \sim v \), the total velocity, at auroral heights it is perhaps adequate to take

\[ v_d = \frac{3v_1 \rho}{2r} \]  

where \( r \) is the distance in centimeters from the earth’s center, \( \rho \) the radius of curvature of the particle path. Since \( \rho = mv/He \), equation 6 becomes

\[ v_d = \frac{3mv^2}{2Her} \]  

Taking \( e/m \) to be \( \sim 10^4 \), \( v = 10^5 \), \( H = 0.5 \), \( v_d = 3 \times 10^{19}/6.4 \times 10^{18} = 4.7 \times 10^5 \) cm/sec. During a magnetic bay a typical current strength is \( N \epsilon v_d \) which equals (say) 5 \times 10^4 emu. Note the important point that grad \( F \) and \( v_d \) can change signs locally owing to the electrojet.

Thus

\[ N = \frac{5 \times 10^4}{4.7 \times 10^5 \times 1.6 \times 10^{-6}} \sim 6.7 \times 10^3 \]

If the current cross section \( A \) is 1000 km² = 10²⁶ cm², \( n = N/A = 6.7 \times 10^3 \), probably not

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**Fig. 4.** Contour lines corresponding to equal integral values \( I \), southern hemisphere, geomagnetic coordinates.

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**Legend**

\[ F = 0.45 \text{ cgs} \]
\[ F = 0.50 \text{ cgs} \]
\[ F = 0.20 \text{ cgs} \]
\[ F = 0.50 \text{ cgs} \]
\[ 1 = 0.7 \text{ cm/sec} \]
an unreasonable value. For an auroral arc 5 km thick and 200 km high (say), only a modest supply of charge is necessary. At low levels the drift is much reduced by collisions, but even so a downward flux of $10^8$ cm$^{-2}$ sec$^{-1}$ is possibly adequate to contribute significantly. One possibility is that the net average westward electric current flowing along the auroral zone, deduced for bays averaged on magnetically disturbed days as compared with quiet days, might be contributed to by this mechanism.

Winds may drive the ionized air beneath the auroral arc and also contribute to the generation of currents, as has often been suggested. Another effect will be change in height of mirror point due to local additions and subtractions of the magnetic fields of the drifting currents involved. It would seem desirable to examine these various possibilities more fully, but the drift current hypothesis for bays would appear especially to warrant further study. Finally, electrical forces owing to unequal penetration of the atmosphere by the spiraling particles, and the different latitude of arrival of positive and negative particles, will yield vertical electric fields (and fields transverse to lines of equal $I$) driving current approximately in the direction $F \times \text{grad } F$.

According to these views, simultaneous bays in the northern and southern hemispheres may occur, but it will be of unequal strengths. There may also be effects of downward (or upward) drifts $v_n$ in response to local anomalies. Thus the mirror points $F_m$ supposedly indicated in Figure 1 at ends of field lines are not at the same heights in the two hemispheres. For this reason also, auroras may not always occur simultaneously in both hemispheres in some areas. An example of such special regions may be those near Australia and central Alaska, in view of considerable differences in height of mirror points.

Integral invariants of auroral particle motion. Alfvén and others [Alfvén, 1939, 1950, 1955; Welch and Whitaker, 1959; Rosenbluth and Longmire, 1957] have indicated that a spiraling particle can be regarded as having a magnetic moment, a useful invariant of the particle mo-

Fig. 5. (A) Curve of equal integral invariant for $I = 15.7$, on surface $F = 0.50$ cgs, heights in km; (B) projection of (A) along field lines to height 100 km, and corresponding southern conjugates (A') and (B').
tion for particles of energies similar to those usually discussed in connection with the aurora. In fact since

\[ v^2 = v^2 - v_1^2 \]  
(8)

using equation 2, and the concept of magnetic moment \( \mu \) being given by the product of the area perpendicular to \( F \), by the current perpendicular to \( F \) is proportional to \( v_i^2/F \), or from equation 8, and \( v_i^2/F = v'/F_m \).

\[ v_2^2 = (1 - F/F_m)v_1^2 \]  
(9)

On the surface \( F_m = \text{constant} \), there can then be defined an invariant

\[ I = \int v_2 \, dl/v \]

\[ = \int (1 - F/F_m)^{1/2} \, dl \]  
(10)

the path of integration being along a line of force \( l \) from the point where the line rises above the surface \( F_m = \text{constant} \) in the southern hemisphere to the corresponding point in the northern hemisphere. Then as the particle drifts in accord with equation 3 at auroral levels, it should on this basis also drift to a line such that \( I \) remains approximately constant [Welch and Whitaker, 1959; Van Allen, McIlwain, and Ludwig, 1959b; Rosenbluth and Longmire, 1957] and dominating for interhemispherical motion.

Figures 3 and 4 give the results of evaluations of \( I \) obtained using the 48 coefficients of the geomagnetic field for 1955.0. The lines of equal \( I \) appear to describe very well the contours of the isochasms derived from auroral observation, except near Hudson Bay. The correction suggested by Hultqvist, if adopted, and taking \( I = 15.7 \) to represent the average isochasm of maximum auroral frequency dictated by the earth’s main field, the discrepancy noted between the observed and computed southern auroral zone is reduced. This assumes interlinkage of the northern and southern auroral zones by lines of force [Vestine and Sibley, 1959]. The curves \( I = 15.7 \) are shown in Figures 5 and 8 with the height in kilometers to the surface \( F = 0.50 \). These heights range from 57 km to 420 km in the northern hemisphere and from -15 km to 633 km in the southern hemisphere. Since aurora most often occur near a height of 100 km, points on the curve \( I = 15.7 \) have been considered to drift along the geomagnetic field lines to the 100-km level to points indicated by dotted points. The new curve (\( I = 15.7 \), reduced) at the 100-km level may fit the observed data slightly better. This experiment shows, however, that the contours so revised change by only a degree.

Fig. 6. Field values in cgs along (B) and (B') of Fig. 5, on the basis 48 coefficients, 1955.0.
or two of latitude. The fit with the new and recent IGY data beginning to become available appears to be better near the geographic south pole than in Figure 1.

Figure 6 indicates the curves at the 100-km level on magnetic field lines intersecting the curves \( I = 15.7 \) at the surface \( F = 0.50 \). The values of total intensity \( F \) at the 100 km level are also shown in either hemisphere. It will be seen that on dropping from the surface \( F = 0.50 \), from points along the contour \( I = 15.7 \), the appropriate mirror points change in the case of any particles leaking to the 100 km level. The new values of \( F \) range from 0.491 to 0.569 cgs in the northern hemisphere, and from 0.470 to 0.668 cgs in the southern hemisphere. This raises the interesting question whether some auroral regions, northern and southern, may be more likely to evince similarity in the incidence of aurora than others.

**Reflection points affected by auroral electrojets.** Along the zone of maximum auroral frequency flow intense overhead electric currents with a field pattern at ground level often described as resembling that resulting from an overhead linear current. If the height is about 100 km, since the field \( f \) of a linear current is given by

\[
f = 2i/r
\]

where \( i \) is the current and \( r \) the perpendicular distance from the current, this current will contribute a field additional to that of the earth's main field \( F \) at auroral levels. If we take a westward flowing current at the northern auroral zone and a direction of \( r \) perpendicular to the main field \( F \), north of the current the field will be \( F + f \), and south of the current it will be \( F - f \). If \( i \) is directed eastwards the situation will be reversed.

In early morning hours in the northern hemisphere, \( i \) is directed to the west. Then the mirror points for particles should be higher above the ground to the north of the electrojet, and lower to the south of the electrojet. It is not yet known what are the consequences of change in mirror point with the position and strength of the current \( i \). One of the simplest possibilities is that in the case of a westward-directed northern electrojet, the current at times may tend to drift to the south, due to enhanced ionization produced by particles mirroring to a lower level to the south at such times. Conversely, in the case of an eastward-directed electrojet, the tendency may be reversed. This may be a possible explanation of the effect of this kind noted by Harang [1951].

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**Fig. 7.** Maximum heights above earth's surface of lines conjugate at the average (observed) auroral zone given by Vestine (I), at the (calculated) theoretical auroral zone given by \( I = 15.7 \) for surface \( F = 0.5 \) cgs (II), and for observed isochasm 40 per cent frequency (III), in earth radii, equatorial plane.
This effect of the electrojets upon the local height of mirror point for auroral particles will be likely also to affect the morphology of auroral displays, as well as the local ion content of the ionosphere.

Further applications of the integral invariant concept to the physics of the aurora and to polar disturbances have been discussed in another recent study by Vestine [1960]. It was suggested that the appearance of eastward-directed electrojets at the auroral zone above conjugate points at the auroral zone was due to penetration below or to the E region by temporarily trapped particles of small pitch angle in the geomagnetic field. The influx of particles was supposed to take place high above the earth where, owing to small scale breaking up of the solar stream and some roughening of the field lines, rapid penetration might occur, as has been suggested [Dessler and Parker, 1959; Piddington, 1960]. Since positively charged particles would drift west and electrons east, the incoming gas would be correctly polarized at some level in or below the E region, and might suffice to drive the eastward-directed electrojet on the afternoon or early evening side of the earth. The particles of large pitch angle would meanwhile drift around to the night side of the earth, where the field lines in space might be less compressed than on the sunward side, or even stretched by the stream. Integral invariant property would then dictate drift of these particles more equatorward, with their mirror points somewhat deeper in the atmosphere. Consequently some of them might again polarize the low ionosphere and the air below to drive the larger westward-directed electrojet, and produce aurora and radio blackouts. Unfortunately, evidence of such distortion of the geomagnetic field has not yet been reported during the world magnetic surveys by earth satellite, but if solar streams do provide such distortions, the pat-
terns of radio blackouts and more complex auroral forms might be explained.

Actual calculation of the charge required to drive the electrojets is difficult, but a voltage of the order of a few tenths of a millivolt would probably suffice. The driving force will be along, and probably also perpendicular to, the auroral zone, perhaps depending upon the unstable drainage of the temporarily trapped auroral particles into the Van Allen radiation belt, or, in any event, within the geomagnetic field. Such drainage of auroral particles has in fact been suggested by Van Allen and his co-workers.

Stability of auroral displays. The auroral displays imagined to arise from temporarily trapped radiation in unstable condition draining into the northern and southern auroral zones, during night hours, must have a stability adequate to maintain the position and charge supply for periods of order minutes and even hours. Since some differential penetration of ions with respect to electrons will occur, the electrojet or auroral-forming plasma will be subject to electrical fields tending to disrupt the display. These electric fields will build up more slowly if the electrojet forming plasma is anchored in the electrically conducting E region. In fact, the first part of an auroral display, after a glow, may be associated with increase of electrical conductivity of the E region, permitting the homogeneous arc which follows to remain quiescent and steady for some time. With rise in the electrical forces, associated also with exponential growth in time of irregularities or flutes in the display, ray arcs may form. These are anchored in higher, less-conducting levels of the atmosphere, so that their duration is transitory, ending in an instability disrupting the display. Dissipation of the disrupting forces may then permit the sequence of auroral forms to recur in the same locality, tending to lead to a typical auroral morphology associated with the growth of instabilities, viz., homogeneous arc → ray arc → rays and draperies → corona → pulsating patches.

Intersection of geomagnetic field lines with the equatorial plane. The geomagnetic lines of force interlinking the auroral zones shown in Figure 1 intersect the equatorial plane at heights ranging from about 5.6 earth radii above geographic longitude 21°E to 5.8 earth radii at 129°E, it may be difficult to note any asymmetry in the average geomagnetic storm-time variation with longitude; note that distances are 6.6 and 6.8 earth radii from the earth's center.

The lines of magnetic force interlinking the computed theoretical average auroral zones therefore intersect the equatorial plane at a distance nearly 7 earth radii from the earth's center. At this distance the geomagnetic intensity is then of order 0.3/7°, if 0.3 be taken roughly as the equatorial surface value of field, or about $8.8 \times 10^{-4}$ cgs. For an electron the spiral radius is then $R = m v/F_e$ where $v$ is about $10^9$ cm/sec, or $R_e$ is about 0.7 km. In the same way $R_p$ is about 1200 km for a proton. If, as suggested above, the magnetic lines of force of a solar stream and those of the geomagnetic field are parallel in some areas of the surface of contact near the equatorial plane (for instance, near the dawn and evening meridians), spiraling particles may be able to transfer from the solar stream to the geomagnetic field. The particles would be unable to distinguish the set of lines of force of the solar stream from those of the geomagnetic field. The reduction in $R$ to auroral levels $F = 0.5$ (say) will then be by a factor of about $8 \times 10^{-7} / 0.5$ or $1.6 \times 10^{-4}$, so that $R_e$ is about 1 meter, with the value $R_p$ for the proton equal to about 2 km, if $v = 10^9$ cm/sec. Although the transition region between the solar stream and geomagnetic field may be of the order of a mean free path, this region may be so irregular in structure that the particles therein become too scattered to form a coherent auroral display. Those penetrating the geomagnetic field ordinarily will be unable to penetrate to a distance of a free path. Consequently, a thin sheet of spiraling particles may appear
more likely, so that homogeneous auroral arcs, relatively thin in the north-south direction result. This seems to be a matter of considerable theoretical interest and warrants careful examination.

In spite of numerous references above to the drift of spiralling auroral particles in the direction $\mathbf{F} \times \nabla F$, or speaking more precisely, parallel to curves of equal $I$, it would be remiss here not to cite considerable evidence which may be incompatible. There may, for instance, be effects of added influences such as those of associated electric fields. Thus there are homogeneous arcs and ray arcs whose eastern (or western) end may terminate in longitudinal extent and remain in this form for minutes, even hours. A rather abrupt rise in the height of mirror point above the east-west extent of the display may be compatible with the drift, explained by some electric fields related to the electrojets, that may have been formed by the display. There are also numerous instances of

![Fig. 9. Approximate intersections of lines of force of the geomagnetic field with the earth's surface, northern and southern hemispheres, based on first 48 Gauss coefficients.](image)
homogeneous arcs in pairs joined at either end, suggesting a westward drift at one latitude coupled at the eastern or western ends of the arc to an eastward drift in the adjacent arc. In this event there could also arise $E \times F$ forces, with $E$ directed nearly north-south, tending to promote drift of spiraling particles in an east-west direction. If $E$ in the space near the ends of the arc were to the west, the incoming or up-going particles should be driven equatorward or injected to lower levels of the outer Van Allen region. Although the aurora and electrojet have been thought of as owing to drainage of particles from the contact region between solar streams and the geomagnetic field, or from a temporarily trapped supply of particles in the outer Van Allen region, there may be loading of the outer Van Allen region by a part of the supply if $E$ is directed from east to west, at the ends of an arc. It would be interesting to examine a few of these questions with the aid of rockets carrying counters and magnetic or electric sensing devices.

Figures 5 and 6 probably afford the best computed estimates of the theoretical average auroral zones in the present paper. The only assumptions made are that the geomagnetic field is given by Finch and Leaton’s coefficients, and that the drift accords with the integral invariant principle of Longmire and Rosenbluth. Assuming then that a charged particle is reflected at the surface $F = 0.5$ cgs at height 100 km just north of Tromsö, Norway (where the observed auroral zone is well determined), curves of equal $I = 15.7$ units were computed. These are shown for the northern and southern hemispheres. Instead of using graphical interpolation of computed values of $I$ to obtain values of equal $I$ shown in Figures 3 and 4, the values at the points shown are those computed directly by machine to be $I = 15.7$ units. It will be noted also that the heights of conjugate mirror points seldom vary by more than 200 km. Other results relating to conjugate points are summarized in Table 1 and Figure 9.

Since the foregoing work was undertaken an interesting paper by Hultqvist [1958] has come to our notice. Using the spherical harmonic analysis of Vestine and Lange for 1945 he gives polar charts showing the corrections to be applied for conjugate points derived from the dipole term, if all coefficients up to and including those for $P_{55}$ ($\cos \theta$) had been used. In general, these results show good agreement with those of the present paper. He also shows the corrections to be applied to conjugate point position if the series expressing the magnetic potential is assumed to terminate with the quadrupole terms. A theory on adiabatic invariants has also appeared [Northrop and Teller, 1960].

Conclusions. The average isochasms of maximum auroral frequency in the northern and southern hemispheres appear to be more or less closely interlinked by geomagnetic field lines. This interlinkage was studied using the first 48 coefficients in the spherical harmonic expansion of the earth’s magnetic field obtained by Finch and Leaton. Use of integral invariants demon-

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</tr>
</tbody>
</table>
strates geomagnetic control of averaged auroral arcs over the polar caps.

It seems likely that features of drift of the particles discussed previously by Alfvén [1939, 1950, 1955], Singer [1957], Christofilos [1959], and Welch and Whitaker [1959], can be usefully applied also to explain the horizontal direction of auroral arcs. Because these auroral arcs also represent patterns of ionization, the directions of current flow in polar electrojets are associated with the direction, but local magnetic fields of flowing currents may produce major departures in the direction of current flow.

The calculated intersections with the equatorial plane of the geomagnetic field lines interlinking the observed auroral zones form a fairly regular oval varying in distance above the earth from 5.6 to 5.8 earth radii.

Acknowledgments. It is a pleasure to acknowledge the skilled assistance of Patricia A. Walters, Isabelle Lange, and George Margadonna in preparing the table and diagrams.

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(Manuscript received April 14, 1960.)