On the Position of the Focus of the Geomagnetic $S_q$ Current System

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Abstract. This paper reviews present knowledge about the $S_q$ foci, discusses the cause of changes in the $S_q$ field, and describes problems concerning the determination of the $S_q$ focus. It is concluded that the foci in the potential fields of geomagnetic $D_s$ variations may be taken as approximately indicating the positions of ionospheric $S_q$ current foci.

1. The Focus of the $S_q$ Current System

The term 'S$_q$ focus' may be defined in more than one way. Ionospheric workers use it to mean the daytime center of circulation of the overhead electric current system that is the primary cause of the $S_q$ (quiet-day solar daily) magnetic variation. This varying current system induces electric currents within the earth, which also contribute to the $S_q$ magnetic variations observed at the earth's surface. From the horizontal components of this variation the magnetic potential $V$ of the total $S_q$ field at the earth's surface can be determined. There is a point of minimum $S_q$ potential in the northern hemisphere, and a point of maximum potential in the southern hemisphere. These points may be called the $S_q$ potential foci.

The potential $V$ can be expressed at any instant as the sum of a series of spherical harmonic terms. The $S_q$ variation of the vertical magnetic intensity at the same instant can be expressed by another series of spherical harmonic terms. By comparing the two series $V$ can be divided into two parts: $V_e$, the surface magnetic potential of the external electric current system; and $V_i$, the surface magnetic potential of the induced currents. Each part will have maximum and minimum points. From $V_e$ it is possible to draw the current lines of the external system, supposed to flow at some chosen uniform height $h$ above the earth. The positions of the northern and southern current foci will depend only slightly on the value of $h$ for heights corresponding to the $D$, $E$, and $F$ layers. Their radial projections on the earth's surface will be close to the $V_e$ potential foci. (In this case the current lines coincide with the equipotential lines. The current function is defined so that its maximum corresponds to a minimum of the potential.)

In a previous paper [Hasegawa, 1950] I have compared the $V_e$ foci with the $V$ and $V_i$ potential foci, using monthly or annual mean data for $S_q$. The differences of position were not large. The same conclusion may be expected to apply also to the foci determined at particular instants. We may therefore take the $V$ potential foci as approximately indicating the positions of the ionospheric $S_q$ current foci. The $V$ potential foci can be rather accurately determined graphically if recorded $S_q$ data are available for several points suitably distributed along a meridian in the middle latitudes. We may then consider to what extent these potential foci are likely to differ from the ionospheric current foci.

2. Summary of Present Knowledge about the $S_q$ Foci

(a) The average position of the foci. Figure 1, from Chapman and Bartels [1940], shows the current lines of the ionospheric $S_q$ system, drawn by Bartels from Chapman's spherical harmonic analysis (1919) of $S_q$ for the minimum sunspot year 1902. It shows the daytime current foci at 40°N, 11h local time, at the equinoxes (the mean of the months March, April, September, October), and at 30°N, 11.5h, and 45°S, 11h, at the northern summer solstice (mean of the months May to August).

Other analyses of $S_q$, based on data for other times and observatories, have given fairly similar current diagrams.

These analyses were not specially directed to-
ward the accurate determination of the positions of the current foci. For this purpose more observations in middle latitudes, better distributed, were necessary. The Second International Polar Year, 1932–1933, supplied data for 46 observatories between the latitudes 60°N and 60°S. But even this was not quite adequate to determine the current foci accurately at individual epochs, especially in the southern hemisphere. However, the attempt was made to determine the most probable values of the $S_q$ field vector over the belt of the earth’s surface between 60°N and 60°S by taking account of the continuity of change of the $S_q$ field as it progresses around the earth with respect to universal time (UT). For the mean of the summer and the winter solstice months twelve $S_q$ current diagrams were obtained, for the epochs 0, 2, 4, ... UT, that is, for twelve uniformly spaced aspects of the earth relative to the sun [Hasegawa, 1950]. The potential was calculated by numerical integration. At epochs of UT at which several observatories are suitably located relative to a focus, the positions of the foci on these maps should be reasonably accurate. Figure 2 shows the (geographical) latitudes of the foci at the twelve UT epochs, estimated from the sequence of maps. The points associated with the northern focus are ringed by a full circle; those for the less-well-determined southern focus, by a broken circle. Figure 3 shows the corresponding ($V$) equipotential lines for the $S_q$ field averaged over the twelve UT epochs; the coordinates of the base map are geomagnetic. The north daytime ($V$) focus is shown to be at 35°N, 11.0h, and the southern one at 35°S, 11.3h. These latitudes and local times are not necessarily the means of the
twelve sets of values for the foci at the individual UT epochs, because the general intensity of the total surface potential $V$ varies with UT in the course of a day; for 1932-1933 (solstice) the ratio maximum to minimum intensity was more than 3–2. In such diagrams as Figures 1 and 3, the mean positions of the foci will be especially influenced by the positions at the epoch of maximum $V$. Moreover, local peaks or other irregularities in the distribution of $V$, which influence the positions of the foci at particular UT's, will be smoothed out in the average $S_4$ days.

In Figure 2 the focal latitudes are shown by points (on the map). The actual potential at each UT epoch is shown by a number along each point.

The north focus travels westward from North America across the Pacific Ocean at approximately 31°N (geographic): the latitude increases to a maximum in middle Asia, and, maintaining the average value of about 38°N over the Atlantic Ocean, declines to 31°N in North America; the average geographic latitude is about 34°N. The south focus is mostly in a somewhat higher latitude than the north focus, the average being about 36°S, but its position is less well determined. In both hemispheres, however, magnetic observatories adequate to determine the tracks of the foci are lacking. Only in five regions (Gulf of Mexico, South America, West Pacific, South Australia, and the Mediterranean region) are there adjacent observatories on both sides of the track, and even they are not most suitably located. The most favorable zones, which may be called European, Far Eastern, and American, give the following focal latitudes:

<table>
<thead>
<tr>
<th>Latitudes of the $S_4$ potential foci</th>
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<tbody>
<tr>
<td>Zone:</td>
</tr>
<tr>
<td>North focus</td>
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<tr>
<td>South focus</td>
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As regards the local time or position in longi-
tude (relative to the meridian through the sun), the north focus, according to Figure 2, may be situated as early as about 1 hour before noon, and the south focus may be situated some 1/3 hour later than the north focus. But in view of the paucity of observatories suitably disposed near the focal tracks, these local times are rather uncertain, and there is need for further study based on better data.

It seems likely that on magnetically quiet days the focal latitudes can be estimated with fair accuracy by considering the $S_q$ value of the horizontal intensity $H$ on the meridian of 11h.

For comparison with the above results for $S_q$, a diagram, Figure 4, is taken from Namikawa [1957], showing the equipotential lines of the $S_q$ field referring to the Second Polar Year (sunspot minimum) as the mean of the summer and winter solstice months. It is interesting to see that the positions of the daytime foci of $S_q$ do not differ from those of $S_q$ (Fig. 3) in the mean state. We will discuss this point later, in section 5(c).

(b) The daily variation of intensity of the $S_q$ field. As the earth turns relative to the sun, there is a daily variation of the general intensity as well as of the form of the $S_q$ field. A convenient general measure of the intensity is the value of the minimum or maximum potential at the north and south foci respectively. In the northern hemisphere the greatest numerical value of $V$ is attained at 14h 16h UT, and the maximum potential at the south focus is numerically least around that epoch. The potential difference between the two foci has a smaller daily variation, as if the change at the south focus nearly compensates for that at the north focus. This might be expected, as the $S_q$ field will be largely controlled by the direction of the geomagnetic axis as this rotates with the earth.

The daily UT change of energy of the $S_q$ field is greater than its mean energy.

(c) The day-to-day $S_q$ changes. The $S_q$ field undergoes active and complicated changes, from day to day, which as yet have been but little studied. Bartels has given harmonic dials showing the daily dial vectors for the first and second components of $S_q(H)$ at Watheroo (not far from the south focal latitude). Ota [1949, 1950] has studied the north focal latitude in the Far Eastern zone for each day of the Second International Polar Year. The determination of the focal latitude on magnetically disturbed days requires that allowance be made for the presence of the additional disturbance field. This, not an easy problem, is discussed in section 5(c). Even during quiet periods the focal latitude may change considerably from one day to the next. Figures 5 and 6 illustrate this; they show the $S_q$ equipotential lines (Fig. 5) and the external $S_q$ current lines (Fig. 6) at 11h local time (2h UT) in the Far Eastern zone on June 21, 1934. Figures 7 and 8 are corresponding diagrams for the same hours on June 23, 1934. Both days were magnetically quiet, the daily character figures $C$ being zero. On June 21 the main north focus was at $28^\circ$N, 11.2h; the weak south focus was at approximately $36^\circ$S, 12h. Forty-eight hours later the two foci were further north by about $15^\circ$ (north focus) and $12^\circ$ (south focus). At the observatories between the two positions of the

![Fig. 4. Similar to Figure 3, but for the $S_q$ field. The symbol shows the position of the Sun.](image-url)
north focus, the type of the $S_q$ ($H$) variations was quite different on the two days.

Figures 5 to 8 also indicate that the latitude of the external current focus may differ from that of the corresponding potential focus by 3° to 5°. It is hardly possible at present to judge which focus is the better for the discussion of the changes of ionospheric current focus or of detailed irregularities in the ionospheric current system. The position of the current focus cannot at present be determined with an accuracy better than 2° to 3°. The factors on which the accuracy depends are discussed in section 5.

Figure 9 shows the difference between the overhead current systems (Figs. 6 and 8) on June 23 and June 21, 1934. Figure 10 is a corresponding diagram showing the difference as between September 24 and 23, 1933. The cause of such day-to-day changes is discussed in section 3.

(d) The seasonal change of latitude of the $S_q$ potential focus. Ota used the magnetic records of the Second International Polar Year to study the seasonal change of $S_q$ focal latitude in the west Pacific region, which is the most favorable zone as regards the number and location of observatories suitable for the purpose. There he determined the north focal latitude for each day during the 13-month period. Omitting uncertain values, he calculated 30-day means to show the seasonal variation of the focal latitude, with the result shown in Figure 11. This indicates a marked seasonal variation with a range of about 10°—the focus is farthest north (about 39°) when the sun is farthest
south, but from March to September it varies only slightly around a value of about 31°. It seems likely that the south focus has an approximately corresponding variation, being most southerly around June and July.

Ota attempted to determine the seasonal variation of the focal latitudes for other zones also, with only moderate success. Further study of the seasonal variation is needed, with better data.

3. The Cause of the Changes in the $S_q$ Field

The $S_q$ field changes from day to day [section 2(c)] and seasonally [section 2(d)]. It is a product of dynamo action by winds in the ionosphere; the distribution of the currents produced by the dynamo emf's depends on the distribution of ionospheric ionization and the resulting conductivity. Ionospheric observations show that there is a considerable seasonal change of ionization, which must greatly affect the $S_q$ current system. From the geomagnetic observations, Maeda [1955; 1957] and Kato [1956; 1957] inferred the ionospheric wind distribution at the solstices and equinoxes, taking account of the anisotropic conductivity. They concluded that at the solstices the wind flows across the equator in opposite directions in different longitudes. The cause of these wind changes must be dynamic (aeronomic or meteorological), not electromagnetic.

The day-to-day changes in the $S_q$ field seem likely to be due to wind changes rather than to changes in the distribution of ionization and conductivity. In the spherical harmonic expres-
sions for the difference between current systems of Figures 9 and 10, the presence of symmetrical terms such as $p_1^1$, $p_1^2$, $p_2^2$ is conspicuous, just as in the difference between the summer and winter $S_q$ fields. Both Figures 9 and 10 indicate a notable circulation of electronic current across the equator. It seems likely that this is due to a change in the wind system. Without detailed calculations the nature of the change of the wind system cannot be understood, but it may consist of wind circulation across the equator.

4. THE PRACTICAL DETERMINATION OF THE $S_q$ FOCUS

(a) When a focus is surrounded by several stations its position can be determined as the point from which the $S_q$ horizontal force vectors radiate. Even during the IGY such determination was seldom possible. The northward and southward movements of the focus require a more widespread network for the application of the method than would otherwise be necessary. To determine the focus for a particular UT, on a single day or for monthly or seasonal mean $S_q$, one must often use stations that at the time are as much as 30° to east or west of the focal meridian. The resulting inaccuracy in the estimated focal position is probably reasonably small.

(b) On the focal meridian at any particular UT the potential reaches its extreme value at the focus. It should be calculated from the values of $S_q(X)$ along this meridian at that UT, but, where such values are lacking, values...
of $q(X)$ at 11h local time at adjacent stations within, say, 1 hour difference of longitude may be used instead.

From a curve of $q(X)$ as a function of latitude along the focal meridian the focal latitude is determined as the point where $q(X)$ changes sign through the zero value. To determine this latitude it is desirable to have more than four available values, some on each side of the focus. Sometimes the available adjacent values are fewer, or all on one side of the focus, or do not give a smooth $q(X)$ curve. In such cases Ota [1949, 1950] was aided in drawing the curve by a knowledge of the form of curve of $q(X)$ for the 10h or 12h meridian in the region of the earth on quiet days. Figure 12 shows some of his results, as communicated at the 1948 Oslo Assembly of the International Association of Terrestrial Magnetism and Electricity.

In studying the $q$ focus from the IGY magnetic data the same method can be used. But here, as also in relation to Figure 12, the results are somewhat uncertain on disturbed days [section 5(c)].

5. Problems concerning the Determination of the $q$ Focus

As the $q$ potential focus is at the position of maximum or minimum potential, and as the potential varies slowly near such extreme values, a weak anomaly in the $q$ distribution can materially change the position of the focus. Hence it is desirable to examine the factors that may thus influence the position.

(a) The base value of the $q$ variation. The values of $q(X)$ are usually reckoned from the daily mean value of $X$. But from the standpoint of the dynamo theory another choice of base line is better. The choice proposed by Hasegawa and Maeda [1951] and later by Maeda [1955, 1956] was based on a dynamo calculation of the $q$ currents, taking the daily mean electromotive force at any station to be zero, and eliminating the daily variation of conductivity. The change of base line is denoted by $c_0$ (Fig. 13); it is determined for $X$ and $Y$ separately; $c_0(X)$ and $c_0(Y)$ as a function of latitude, as determined from $X$ and also from $Y$, for summer, winter, and the annual mean. $c_0(X)$ has a maximum at the equator, and changes sign at about 35° N and S, at about the focal latitudes of $q$. The numerical value of the potential at the daytime $q$ focus is increased by about 45 per cent by the change of $X$ base line by the amount $c_0$.

Figure 2 shows two broken lines, lying nearly along the 35° latitude circles, which indicate the latitudes at which $a$, the first harmonic coefficient of $q(X)$, changes sign.

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**Fig. 11. Seasonal variation of the northern focal latitude in the west Pacific region during the Second Polar Year.**

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**Fig. 12. An example showing the day-to-day focal positions (part of the Second Polar year data).**

- $N_{10}$, $N_{11}$: North focal positions determined by the $X$ curves along the 10h and 11h meridians, respectively.
- $S_{10}$, $S_{11}$: South focal positions determined by the $X$ curves along the 11h and 12h meridians, respectively.
- $N_{10} + S_{11}$: The distance between the foci $N_{10}$ and $S_{11}$ along the meridian.
- $N_{11} + S_{10}$: The distance between the foci $N_{11}$ and $S_{10}$ along the meridian.

The open circles indicate less accurate values.
It was suggested that in determining the focal position the base value for $X$ should be the right value, or the mean of the sunrise and sunset values. In some cases the base value can be read directly from the records, but in middle latitudes this is not easy. Ota is attempting to determine the day-to-day variations of the focus from the IGY data, using as base value the mean of the sunset and sunrise $X$ data. His preliminary report is given in the Appendix.

(b) The effect of the internal current system.

1. The general structure of the crust and the interior of the earth. In the main, the principal spherical harmonic terms in the external and internal parts $V_x, V_y$ of the surface magnetic potential hold similar ratios, and the differences of phase angle, exterior-interior, are only a few degrees. But the relations are less simple for the higher harmonic terms. This departure from similarity may be ascribed to several contributory causes. It may depend partly on the insufficiency and unsatisfactory distribution of the magnetic data, but it may also depend on the nature of the earth's crust and deeper interior. Though it is not unlikely that the $V_x$ and $V_y$ potential foci agree within about 3°, it is desirable to make new analyses of $V$ for $S_x$ into its parts, $V_x$ and $v_x$, on the basis of the IGY data.

2. Inequalities within the crust. Any irregular distribution of electrical conductivity within the crust will produce a local anomaly in $S_x$. World-wide studies of $S_x$ average out such local anomalies, but they may affect the determination of the $S_x$ foci at particular epochs of UT. However, as the anomalies seem to affect $S_x$ more in $Z$ than in $X$ and $Y$, their influence on the focal positions may be small.

3. The distribution of land and ocean. The electrical conductivity of a deep ocean will be sufficient to shield the underlying crust from the external varying $S_x$ field, so that some induced currents flow in the ocean. Hence $V_x$ may be expected to vary with UT, owing to the irregular distribution of continents and oceans. The regions of the earth may be roughly classified into continental and ocean hemispheres, and some of the variations of latitude shown on Figure 2 may depend on that distinction.

Sometimes we have a twin structure of the northern focus in the Far East region. Figure 14 shows the chart of the magnetic potential of $S_x$ on September 23, 1933, obtained by means of the method of graphical integration. In this chart we had two foci between 10h and 11h, deduced from observatories in this region. At three stations, Kakioka, Aso, and Zō Sēs, we had records of $H$ variations of a typical equatorial type, showing that the observatories passed south of the focus (say focus I) while...
the horizontal vectors of the variations described a clockwise rotation, which is characteristic of the daily variations at observatories north of the focus (say focus II).

As the result of calculations made with all available data of the world, the foci of \( V \) and also of \( V_s \) were found to be nearly on the same latitude as focus II, while the focus of \( V_s \) was found to be at the same latitude as focus I, farther north. In other words, focus II is the ordinary one, and focus I is that for the interior electric current. Over the land area to the west, 4 hours later, we found conditions simpler, with a single focus passing between Alibag and Dehra Dun, along the same latitude as focus II. But we cannot be certain that the twin structure of the focus is an effect of the distribution of land and ocean until we examine the effect further by carrying out the analysis of the \( S_s \) field not only in this region but also in other parts of the earth.

(c) The effect of disturbance fields. The \( S_s \) variation of the month is usually defined as the mean of the daily variations on the five most quiet days; therefore it always contains, to some extent, the disturbance variations.

If the electric currents responsible for the \( SD \) and the \( S_s \) variations flow in the same layer of the upper atmosphere, the foci will be formed in the combined electric current system, which is represented by \( V_s \) of the \( S \) field. We expect that electric currents parallel to the meridians near the focal meridian due to the \( SD \) fields will advance or retard the real foci in longitude, but the latitude effect may be small. To determine the position of focus, the effect of the \( Dst \) field on the potential fields must be eliminated. The steady \( Dst \) and the steadily changing \( Dst \) are eliminated by the ordinary method of obtaining \( S \) (solar daily variation) field; only irregular changes of the \( Dst \) fields in the course of the day influence the potential of the \( S \) field.

In illustrating day-to-day changes of the position of the focus, we find many large departures from the average value during magnetic storms. Except for these extreme cases the effect of \( Dst \) on the position of the focus in the monthly or annual mean may be negligible, because the positive and the negative effects seem equally probable. (See Fig. 4.)

The same can be said about the effect of other irregular disturbance fields.

6. Concluding Remarks

There are many unsolved problems. An essential difficulty is the insufficient and unsatisfactory distribution of observatories in middle latitudes; most other problems are unsolved because we have not worked on them. After examining all conditions, we reach the conclusion that the positions of the foci in the equivalent current system can be adopted as an indication for the foci of the overhead electric current system so far as the mean state of \( S_s \) and the well-calculated \( S_s \) on selected days are concerned. As for the day-to-day positions of foci, calculated by the practical method described above, including disturbed days, they may be regarded as revealing the relative variability of the \( S_s \) field. They also provide elementary and primary data for studying the various problems in this field.

Kyoto WDCG for Geomagnetism is attempting to issue the day-to-day position of the foci of equivalent current system of geomagnetic daily variations, if sufficient data are available. We desire the cooperation of all researchers who are interested in this subject, by carrying out
Fig. 15. The day-to-day position of the north focus of $S$ in the Far East region from July 1 to December 31, 1957. Geographic coordinates. Circle, normal; triangle, inexact; $x$, extrapolated; $s$, no value, magnetic storm.

Available observatories: C034 Memambetsu, C147 Kakioka, C214 Shimosato, C223 Aso and E553 Muntinlupa.

similar calculations with data in various regions of the world.

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APPENDIX

A preliminary report on the day-to-day positions of the north focus of $S$ in the Far East Region, by M. Ota. As the base value of the $X$ component, we adopted the mean of the sunrise and the sunset values, which are taken to be the average of $X$ during 3 hours centered at ground sunrise and sunset. Between the main minimum and main maximum points in the $Y$ curve, we determined a point which has the mean of the both extreme values. The time of that point was regarded as showing the focal longitude.

References

Hasegawa, M., Transactions Oslo Meeting, 1948, IATME Bull., 13, 1950. (Refer to the abbreviated diagrams on p. 431.)

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