Differential Rotation of the Magnetospheric Plasma as Cause of the Svalgaard-Mansurov Effect

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Svalgaard, and independently Mansurov, discovered a correspondence between the geomagnetic variations at the geomagnetic poles and the sector polarity of the interplanetary magnetic field (IMF). Heppner noted an asymmetry in the magnetospheric electric convection field observed within the polar caps at ionospheric altitudes which is also related to the sector polarity of the IMF. It is shown that both effects can be consistently explained by differential rotation of the magnetospheric plasma with respect to the earth. If the azimuthal component of the IMF is directed toward dusk, the magnetospheric plasma is slowed down in the northern polar cap and is accelerated in the southern polar cap. If the azimuthal component of the IMF is directed toward dawn, the situation is reversed. This deviation from corotation of the magnetospheric plasma disappears at lower latitudes. It is suggested that this differential rotation is generated by a vertical component of the interplanetary electric field which penetrates into the day side regions of the magnetosphere. A quantitative model of the electric field and current configuration within the magnetosphere and the ionosphere representing these effects is given.

INTRODUCTION

Svalgaard [1968], and independently Mansurov [1969], discovered a correspondence between the geomagnetic variations at the geomagnetic poles and the sector polarity of the interplanetary magnetic field (IMF). The magnetic effect within the polar cap regions can be simulated by ionospheric Hall currents encircling the magnetic poles at about 10° polar distance in a clockwise direction as seen by an observer standing on the ground during interplanetary sectors with fields pointing away from the sun and in a counterclockwise direction during toward-sector polarity [Svalgaard, 1973]. The critical component of the IMF is the azimuthal component rather than the sector polarity [Friis-Christensen et al., 1972]. The polar cap magnetic effect is proportional to the azimuthal component, and its response to this component of the IMF is rather quick with a delay time of the order of 30 min or smaller [Wilhelm and Friis-Christensen, 1972; Kawasaki et al., 1973]. The current strength has its maximum during local noon and critical component of the IMF is the azimuthal component, and its response to this component of the IMF is rather quick with a delay time of the order of 30 min or smaller [Wilhelm and Friis-Christensen, 1972; Kawasaki et al., 1973]. The current strength has its maximum during local noon and during summer. When the magnetic pole is on the night side of the earth, the polar cap electric current is absent or weak [Svalgaard, 1973].

While the seasonal variation of the effect can readily be explained by the seasonally varying electric conductivity of the ionosphere, the driving force is very probably an electric field. Wilheim and Friis-Christensen [1972] have suggested that such an electric field is induced by merging of interplanetary and geomagnetic field lines. Jørgensen et al. [1972] and Stern [1973] have also discussed the cause of this effect by using magnetic merging arguments and assuming an open magnetospheric model.

Heppner [1972] discovered a dawn-dusk asymmetry in the high-latitude electric field observed by Ogo 6 at ionospheric heights. This asymmetry is related to the polarity of the IMF. Within the dawn-dusk meridian over the polar cap the maximum amplitude is on the evening side in the northern hemisphere and on the morning side in the southern hemisphere during toward-sector polarity. The situation is reversed during away-sector polarity.

We want to describe quantitatively the Svalgaard-Mansurov effect as well as Heppner's observations in terms of a local time independent electric potential which is related to a deviation from corotation of the polar cap magnetospheric plasma. The origin of that field will be discussed qualitatively, a closed magnetosphere being assumed.

ANALYTIC MODEL OF MAGNETOSPHERIC ELECTRIC FIELDS

Convection field. The existence of a large-scale magnetospheric convection field generally directed from dawn to dusk is now well established [Heppner, 1972; Cauffman and Gurnett, 1972]. Its strength is related to solar activity. However, its configuration seems not to be significantly altered during geomagnetic disturbances. A particular feature is a field reversal near ±75° geomagnetic latitude within the dawn-dusk meridian with maximum field strength on the low-latitude border of the reversal. The convection field decreases rapidly toward lower latitudes. It remains nearly constant over the polar cap.

An analytic model of that convection field has been constructed by Volland [1973]. It is based on the simplified assumption of a coaxial geomagnetic dipole, and it is valid within the near earth region \( L \leq 10 \). Its electric potential in the northern hemisphere during very quiet conditions is given by

\[
\phi_e = \phi_e x \sin \tau \quad x = (r_0/r)^{1/2} \sin \theta / \sin \theta_0 \leq 1
\]

\[
\phi_s = \phi_s x^{-4} \sin \tau \quad x = (r_0/r)^{1/2} \sin \theta / \sin \theta_0 \geq 1
\]

where \((r, \theta)\) are distance from the earth's center and colatitude, \(r_0\) the radius of the dynamo region, \(\theta_0\) the colatitude of the field reversal, \(\tau\) the local time, and \(\phi_s = 8.5\, kv\), the electric potential between earth and magnetopause within the dawn-dusk meridian. During disturbed conditions this electric potential increases in magnitude. Furthermore, it is supplemented by a secondary electric polarization field originating from the enhanced electric conductivity within the auroral ovals. This secondary potential increases the primary electric field at low latitudes and reduces it within the auroral ovals. It also shifts the location of the plasmapause into the evening sector. This modification is, however, of minor importance for our study and will be neglected here.

Note that Volland [1973] used alternatively the power law \(x^2\) in (1) at high latitudes \(x \leq 1\) in order to fit Gurnett's data.
The situation is antisymmetric with respect to the poles. However, as Heppner [1972] stated, the electric convection field possesses generally finite amplitudes at the poles, so that the power law $x^l$ in (1) is more appropriate.

The meridional component of the electric field in (1) positive toward south is then

$$E_y = -\phi_y x_0 \cot \theta \sin r/r \quad x \leq 1$$
$$E_y = 4\phi_y x_0^{-4} \cot \theta \sin r/r \quad x \geq 1$$

The zonal component positive toward east is

$$E_x = -\phi_x x_0 \csc \theta \cos r/r \quad x \leq 1$$
$$E_x = -\phi_x x_0^{-4} \csc \theta \cos r/r \quad x \geq 1$$

Figure 1 shows the meridional component $E_y$ during dawn-dusk at ionospheric altitudes ($r = r_o$) on the northern and on the southern hemisphere. Note that according to (2), $E_y$ is positive toward south and reverses sign when it crosses the poles. Therefore we plotted $-E_y$ on the dusk side in Figure 1, so that Figure 1 represents the field as it will actually be measured. From Figure 1 we see immediately the field reversal at $15^\circ$ colatitude, the nearly constant amplitudes over the polar caps (the dashed lines), and the strong decay toward lower latitudes. Since the convection field is symmetric with respect to the equator, it is on the southern hemisphere:

$$\phi_x(\pi - \theta) = \phi_x(\theta)$$

Thus $E_y$ changes sign on the southern hemisphere, as is indicated in Figure 1.

The zonal component $E_x$ has maximum amplitudes during noon-midnight and over the polar caps, where its magnitude equals that of $E_y$. At lower latitudes it is 4 times smaller than $E_y$.

The field configuration of $E_y$ in Figure 1 is in general agreement with the observations of Heppner [1972] if the sector polarity effects are averaged out. The steepness of the reversal is exaggerated in our model. The configuration of $\phi_y$ in (1) with the equatorial plane of the magnetosphere and within the ionosphere is also consistent with the observed forms of the plasmapause and the geomagnetic $S_x$ current, respectively.

**Electric field due to differential rotation.** Let us assume that for reasons which we shall discuss in the section on the origin of differential rotation the northern polar cap magnetospheric plasma rigidly rotates with an angular velocity different from that of the earth in a region bounded by an $L$ shell $L_1 = \csc^2 \theta$, where $\theta$ is the colatitude of the foot point of the shell within the dynamo region. Furthermore, between shells $L_1$ and $L_2 = \csc^2 \theta$, a transition from superrotation to corotation occurs. Then an observer rotating with the earth would measure an electric field in the northern hemisphere which has the potential

$$\phi_{dr} = \phi_{dr}^0(x_1 x_3 - x^2) \quad 0 \leq x \leq x_1$$
$$\phi_{dr} = \phi_{dr}^0 x_0(x_3 - x)^3/(x_3 - x_1) \quad x_1 \leq x \leq x_2$$

with $x_1 = (\sin \theta_1/\sin \theta_0)$, $x_2 = (\sin \theta_2/\sin \theta_0)$, $x_e = 1/\sin \theta_0$, and $x$ from (1).

The potential (5) and its derivative with respect to $\theta$ are continuous at the boundaries $x_1$ and $x_2$. Furthermore, $\phi_{dr}$ fulfills the condition that the electric field is orthogonal to the geomagnetic dipole field. The meridional component of (5) is

$$E_y = 2\phi_{dr} x_0^2 \cot \theta/r \quad 0 \leq x \leq x_1$$
$$E_y = 2\phi_{dr} x_0 x_3(x_3 - x) \cot \theta/(r(x_3 - x_1)) \quad x_1 \leq x \leq x_2$$
$$E_y = 0 \quad x_2 \leq x \leq x_e$$

If $\phi_{dr}^0 > 0$, the polar cap magnetospheric plasma superrotates with respect to the earth. If $\phi_{dr}^0 < 0$, the rotation is retrograde. If

$$\phi_{dr}^0 = -\phi_{dr}^0 \sin^2 \theta_0 = -6 \text{ kV}$$

where $\phi_{dr}^0 = 90 \text{ kV}$ is the total electric corotation potential, the plasma within the polar cap is fixed in a nonrotating frame of reference, and $\phi_{dr}^0 = 6 \text{ kV}$ means superrotation with an angular velocity twice that of the earth.

Figure 2a shows the potential of (5) plotted versus colatitude at ionospheric heights on the northern hemisphere. Here we used the numbers $\theta_1 = 10^\circ$, $\theta_2 = \theta_0 = 15^\circ$, $r = r_o$, and $\phi_{dr}^0 = 5 \text{ kV}$. Figure 2b shows the meridional electric field from (6) versus colatitude. In Figure 1 we plotted the sum of the two meridional components of $E_y$ from the convection field (2) and from the differential rotation field (6) within the dawn-dusk meridian at ionospheric heights, adopting the same numbers for $\theta_1$ and $\theta_2$ as in Figure 2. However, we assumed antisymmetry with respect to the equator for the potential $\phi_{dr}$:

$$\phi_{dr}(180^\circ - \theta) = -\phi_{dr}(\theta)$$

and therefore

$$E_y(180^\circ - \theta) = E_y(\theta)$$

for the differential rotation field, so that the northern and the southern polar cap regions rotate in an opposite sense.

Comparison with Heppner’s [1972] results indicates that superrotation in the northern hemisphere and retrograde rotation in the southern hemisphere correspond to an azimuthal component of the IMF directed from dusk to dawn, whereas retrograde rotation in the north and superrotation in the south correspond to a component of the IMF directed toward dusk. Since in general a direction of the IMF toward dawn (dusk) is
related to toward- (away-) sector polarity [Friis-Christensen et al., 1972], we shall use these more impressive expressions.

The form of Figure 1 closely resembles Heppner's measurements with maximum amplitudes within the polar caps near 10° colatitude.

**ELECTRIC CURRENTS**

**Hall current.** The electric fields considered in the last section drive electric currents within the ionospheric dynamo region. On the ground, one observes magnetically primarily the Hall component of those currents because the Pedersen currents close within the magnetosphere via field-aligned currents [Fukushirna, 1971; Vasyliunas, 1968]. The electric current system strongly depends on the temporally and spatially varying electric conductivities. For simplicity, we assume constant height-integrated average Pedersen and Hall conductivities of \( \sigma_p = 15 \Omega^{-1} \) and \( \sigma_h = 10 \Omega^{-1} \), respectively. Then the Hall current driven by the differential rotation field possesses only a zonal component

\[
I_h = -\frac{2 \sigma_H}{r_0^3 \sin^2 \theta} \left(2 - 3 \sin^2 \theta\right) \left(r - r_0\right) \sin \theta \quad 0 \leq \theta \leq \theta_0
\]

with \( E_o \) from (6). This current is plotted versus colatitude in Figure 2b (right abscissa). The current encircles the north pole in a westward direction in the case of superrotation and in an eastward direction in the case of retrograde rotation of the polar cap plasma. Together with the result found in the section on an analytic model of magnetospheric electric fields, this is completely consistent with Svalgaard's [1973] findings that in the northern hemisphere a westward current flows during toward polarity and an eastward current flows during away polarity of the IMF. In the southern hemisphere the situation is reversed.

Since the current of (10) can be derived from a stream function, the magnetic effect of this electric current on the ground can easily be determined from standard methods [e.g., Richmond, 1974]. In Figure 3 the solid lines are the horizontal component \( H \) and the vertical component \( Z \) of the magnetic field on the ground generated by the electric current of Figure 2b plotted versus colatitude on the northern hemisphere. The dashed lines in Figure 3 are derived from observations by Svalgaard [1973, Figure 13] showing the average polar cap magnetic effect during toward polarity on the northern hemisphere. Both the measured and the calculated results agree reasonably well in structure. This is true in particular for the locations of the maximum values of \( Z \) and \( H \) and of the zeros of \( Z \).

Concerning the amplitudes we have to take into account the secondary magnetic fields induced within the earth's interior. They enhance the primary horizontal component and reduce the vertical component [e.g., Chapman and Bartels, 1951]. If we would add those induced magnetic fields, the agreement between the amplitudes would become also quite satisfactory.

**Pedersen current.** While the Hall current is confined to the polar cap ionospheric layers, the ionospheric Pedersen current, which has only a meridional component

\[
I_p = \frac{\sigma_p E_o \left(1 - 2 \sin^2 \theta\right) - 2 + 3 \sin^2 \theta}{r_0^2 \sin \theta (1 - x_1) (1 - x_1)} \quad x_1 \leq \theta \leq 1
\]

Figure 4 shows the field-aligned electric current density in the northern hemisphere at ionospheric heights as derived from (13). Positive values are related to flow out of the ionosphere. Because of (9) the direction of the outflow is reversed in the southern hemisphere. Evidently, this current must close within the magnetosphere. However, since the geomagnetic field lines necessarily must close within the magnetosphere in order to satisfy flow continuity. We determine that field-aligned electric current density \( J_0 \) from the equation of flow continuity:

\[
\frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta I_h\right) + \sin \chi J_1 = 0 \quad r = r_0
\]

where \( \chi \) is the angle of inclination of the geomagnetic dipole field. From (11) and (12) it follows that

\[
J_1 \left|_{r=r_0} \right. = \frac{2 \sigma_H}{r_0^3 \sin^2 \theta} \left(2 - 3 \sin^2 \theta\right) \left(r - r_0\right) \sin \theta \quad 0 \leq \theta \leq \theta_0
\]

\[
J_1 \left|_{r=r_0} \right. = \frac{2 \sigma_H}{r_0^3 \sin^2 \theta} \left(r - r_0\right) \sin \theta \left(1 - 2 \sin^2 \theta\right) - 2 + 3 \sin^2 \theta \quad x_1 \leq \theta \leq 1
\]

\[
J_1 \left|_{r=r_0} \right. = 0 \quad 1 \leq \theta \leq x_s
\]
are open within the polar cap regions, we do not expect an electric link between both hemispheres. In any case the Pedersen current is a poloidal current configuration with only minor magnetic effects on the ground.

Our calculations are based on a constant electric conductivity which is, of course, not realistic. At $E$ layer heights the electron density is nearly proportional to $\cos^{-2/3} \chi$, where $\chi$ is the solar zenith angle. Therefore in the summer hemisphere the electron density at 80° latitude varies by a factor of about 2 between noon and midnight. In the winter hemisphere the $E$ layer electron density within the polar cap is rather small with no extreme diurnal variation. Thus the observed diurnal variation of the Svalgaard-Mansurov effect can be explained at least partly by the diurnal variation of the electric conductivity within the dynamo region. Clearly enough, the Pedersen current configuration in reality will deviate significantly from symmetry, as is assumed in our simplified model.

**ORIGIN OF DIFFERENTIAL ROTATION**

In this section we want to speculate about the origin of differential rotation of the polar cap magnetospheric plasma. As is well known, an observer fixed within a geocentric magnetospheric coordinate system would measure an induced electric field within the interplanetary space:

$$E = -v \times B$$  \hspace{1cm} (14)

where $v$ is the velocity of the solar wind and $B$ the IMF. The electric field contains a vertical component if the IMF possesses an azimuthal component. Within away-sector polarity of the IMF, that vertical component is directed toward north. Within toward sectors it is directed toward south (see Figure 5).

At the magnetopause the tangential component of that field must be continuous if space charge effects can be neglected. Therefore it penetrates into the daytime region of the magnetosphere. Within lower latitudes, where the geomagnetic field lines are closed, the tangential component is mainly parallel to the geomagnetic field lines. Because of the high electric parallel conductivity, that field will break down rapidly, a deeper penetration of the interplanetary electric field being prevented. However, within the daytime regions of the polar caps a significant part of this penetrating electric field has a component orthogonal to the geomagnetic field lines. Since those lines are open, superposition of the southern and the northern field is prevented, so that the orthogonal components can be maintained. The high electric parallel conductivity allows these fields to map down into the ionosphere.

The $E \times B$ drift generated by these electric fields within the daytime polar caps adds angular momentum to the already corotating polar cap magnetospheric plasma and eventually causes these regions to rotate prograde in the northern hemisphere and retrograde in the southern hemisphere with respect to the earth's surface in the case of toward-sector polarity of the IMF (see Figure 5). For away polarity the directions of rotation are reversed.

An observer corotating with the earth would then measure a magnetospheric electric field within the polar cap regions originating from such differential rotation which has a southward component during toward polarity and a northward component during away polarity. This is in complete agreement with Heppner's [1972] observations of the electric field along the dawn-dusk meridian and also with Svalgaard's [1973] deduction of the directions of the equivalent electric ionospheric currents.

The maximum electric field due to differential rotation at ionospheric altitudes ($r = r_0$) and for a polar distance of $\theta_i = 10^\circ$ is

$$E = \frac{\phi_{tr}}{r_0 \sin^2 \theta_0} (4 \cos^2 \theta_i + \sin^2 \theta_i)^{1/2} = 4 \text{ mV/m}$$  \hspace{1cm} (15)

if we apply the number $\phi_{tr} = 5 \text{ kV}$ from the section on the electric field due to differential rotation.

The driving interplanetary electric field has the magnitude $E = vB = 5 \text{ mV/m}$ if we use the numbers $v = 500 \text{ km/s}$ and $B = 10 \text{ nT}$. Thus the energy of the solar wind is large enough to generate differential rotation of the magnetospheric polar cap plasma.

It should be mentioned that a merging process between equatorial components of interplanetary and magnetospheric magnetic field lines as proposed by Jørgensen et al. [1972] may also supply the angular momentum appropriate to cause differential rotation.

After this paper was finished, a paper by Leontyev and Lyatsky [1974] came to the attention of the author. There similar calculations of electric fields and currents related with the Svalgaard-Mansurov effect have been performed. The model used by Leontyev and Lyatsky is based on an open
magnetosphere. It excludes polar cap electric currents. In contrast to the present paper, no attempt has been made to compare observations with the calculations quantitatively.

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