Kelvin-Helmholtz Instability and the Semiannual Variation of Geomagnetic Activity

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Kelvin-Helmholtz instability along the flanks of the magnetosphere exhibits a semiannual variation, instability maximums occurring at the equinoxes and instability minimums at the solstices. It is suggested that Kelvin-Helmholtz instabilities, generated at the magnetopause, initiate the modulation of geomagnetic disturbance detected as the semiannual variation of geomagnetic activity. The Kelvin-Helmholtz explanation predicts a universal time variation of geomagnetic disturbance. This prediction is confirmed by several analyses of the geomagnetic activity indices. Since the physical mechanism suggested for the semiannual variation of geomagnetic activity depends on the tilt of the dipole axis, the 'equinoctial' hypothesis is supported.

The semiannual variation of geomagnetic activity is a well-established phenomenon, having maximums near the equinoxes and minimums near the solstices (range ≈15 γ). There have been two views as to the cause of this semiannual variation:

1. Cortie's [1912] 'axial' hypothesis explained that the geomagnetic variation was associated with changes of the earth's heliographic latitude (maximum north heliographic latitude of 7.2° on September 7 and maximum south heliographic latitude of 7.2° on March 6). At the times of these maximums, the earth would be more favorably situated with respect to the solar streams coming from the active regions between 10 and 20 deg north and south heliographic latitude, thereby resulting in increased geomagnetic activity at these times. The axial hypothesis depends basically upon two things: that the solar streams cause geomagnetic activity, and that the solar streams emitted radially are not diffused very much in the interplanetary medium by the time they get to the orbit of the earth.

2. McIntosh's [1959] 'equinoctial' hypothesis, on the other hand, attributes the semiannual variation to the change of the tilt of the dipole axis, the times of greater activity occurring when the dipole is perpendicular to the solar wind flow. The maximum activity occurs, therefore, twice yearly at the equinoxes. The exact physical mechanism is not specified.

There has been a continuing controversy over the correct explanation of the semiannual variation of geomagnetic activity, since the nature of the geomagnetic data makes it difficult to identify the dates of the maximums. Priester and Cattani [1962] favor the axial hypothesis in their analysis of Bartels' [1940] monthly geomagnetic activity data, whereas Bartels [1932, 1963] and recently Shapiro [1969] favor the equinoctial hypothesis in their analysis of the geomagnetic data. Mishin et al. [1961] and Mishina [1967] invoke a more complicated interplay of several factors in explaining their geomagnetic analyses.

The lack of a physical mechanism to adequately explain the cause of the semiannual variation has been the major problem. A knowledge of the mechanism will identify the parameters responsible for the phenomenon and therefore make possible the understanding of the semiannual variation of geomagnetic activity.

**Kelvin-Helmholtz Hypothesis**

It is the purpose of this paper to associate the semiannual variation of geomagnetic activity with the variation of Kelvin-Helmholtz insta-
stability at the boundary of the magnetosphere. It will be shown that the variations of Kelvin-Helmholtz instability are due to the seasonal changes of the orientation of the earth's magnetic dipole with respect to the solar wind flow. It is suggested that instabilities generated at the magnetopause may initiate the geomagnetic disturbance detected by surface magnetic observatories as the semiannual variation of magnetic disturbance.

Frictional or viscous-like interactions between the solar wind and the magnetosphere have been studied quantitatively by Axford [1964] and Parker [1958] on the basis of a sound-wave refraction mechanism and particle scattering by inhomogeneities of the magnetic field, respectively. The Kelvin-Helmholtz phenomenon could possibly be the process establishing the initial conditions in both cases.

The seasonal distribution of magnetospheric substorms may well contribute to the semiannual variation of geomagnetic activity. Akasofu [1968, p. 224] cites ten theories that have been suggested to explain the magnetospheric substorm. In two of these theories, Kelvin-Helmholtz instability might again serve as the intermediate between the solar wind and geomagnetic disturbance. It should not be assumed that the Kelvin-Helmholtz mechanism is exclusive, but rather in competition with other disturbance phenomena.

The Kelvin-Helmholtz instability of the boundary between two magnetohydrodynamic incompressible fluids in relative motion has been studied by Chandrasekhar [1961]. A number of important contributions to the problem have been made by Axford [1960, 1962], Sen [1963, 1964], Fejer [1963, 1964], Talwar [1964, 1965], and Southwood [1968]. For a good review of this subject see Dungey [1968] and Gerwin [1968]. As a first approximation, the instability criterion developed by Chandrasekhar can be applied to the magnetopause, giving

\[ U^2 > \frac{\rho_I}{4 \pi \rho_M} [B_I^2 \cos^2 \psi_x + B_M^2 \cos^2 \psi_M] \]

where \( I \) stands for interplanetary values outside the magnetosphere (magnetosheath values) and \( M \) stands for magnetospheric values. The symbol \( U \) is the stream speed of the solar wind at the magnetopause, and \( \psi \) is the angle between the local stream velocity \( U \) and the magnetic fields.

The symbols \( B \) and \( \rho \) stand for magnetic field intensity and mass density, respectively. It has been assumed that the wave vector of the perturbation is in the \( U \) direction. The presence of parallel components of magnetic field with respect to the solar wind stream at the boundary create a stabilizing influence. If the magnetic fields were either nonexistent or perpendicular to the solar stream, a situation of complete instability would result for wave vectors along \( U \). Parallel components of magnetic fields therefore have a stabilizing influence on the boundary. The angle \( \psi_M \) is a function of three quantities: the point on the magnetopause, the time of year, and the time of day. Some regions of the magnetopause are therefore more susceptible to this instability than others and, in addition, they exhibit a time variation. An illustrative quantitative estimate of the Kelvin-Helmholtz instability criterion is given in the appendix.

It is clear from the Kelvin-Helmholtz inequality that interplanetary fields will contribute to variations of instability. However, since seasonal variations of the interplanetary field have not been observed, any interplanetary field influence on a semiannual variation of instability will be averaged out when several years' data are considered. Short-term variations of the interplanetary magnetic field and their influences on geomagnetic activity, as observed by Fairfield [1967], Schatten and Wilcox [1967], and Wilcox et al. [1967] and explained in terms of interconnection between interplanetary and geomagnetic field lines in the manner of Dungey [1961], are by no means excluded by the present discussion.

We will now associate the semiannual variation of geomagnetic activity with the seasonal variations of the angle \( \psi_M \) in the Kelvin-Helmholtz theory. A sketch of the magnetosphere of the earth is shown in Figure 1. The \( Y-Z \) plane cuts a dawn-dusk cross section through the flanks of the magnetosphere and passes through the center of the earth, which is at the origin of the coordinate system. The solar wind is in the \( X \) direction. The dipole is shown here to be along the \( Z \) axis and perpendicular to the solar wind. This particular orientation of the dipole with respect to the solar wind will occur twice a day during the periods that the earth's rotation axis is perpendicular to the solar wind, i.e. during the equinoxes.
An appropriate rotation of this coordinate system about the $X$ axis (solar wind direction) can always be made so that the dipole is contained in the $X-Z$ plane and the $X-Y$ plane intersects the magnetopause along the flanks.

Figure 2 is a cross-sectional view of the magnetosphere in the $X-Y$ plane looking in the negative $Z$ direction for the orientation of the dipole of Figure 1. Inside the magnetopause all along the dawn and dusk flanks the geomagnetic field lines are in the positive $Z$ direction. A local coordinate system $X', Y; Z'$ ($X'$ in the streaming direction, $Y'$ perpendicular to the magnetopause, $Z' = Z$) can be placed anywhere along the dawn and dusk flanks. Such a coordinate system is shown at the dusk flank in Figure 2. Clearly the angle $\psi_M$ is 90° at the flanks in the plane of symmetry, and the inequality becomes

$$U^2 > \frac{\rho_I + \rho_M}{4\pi \rho_I \rho_M} B_I^2 \cos^2 \psi_I$$

favoring instability. This value of $\psi_M$ occurs at 1030 and 2230 UT (sunrise and sunset at the north geomagnetic pole, respectively). For this case, it is apparent that magnetospheric field lines cannot exert stabilizing influences in the $U$ direction.

The least unstable situation at the flanks during the equinoxes occurs when $\psi_M = 78.5°$ or 101.5°. These values of $\psi_M$ occur at 0430 and 1630 UT (midnight and noon at the north geomagnetic pole, respectively). The condition for instability now becomes

$$U^2 > \frac{\rho_I + \rho_M}{4\pi \rho_I \rho_M} [B_I^2 \cos^2 \psi_I + 0.0397B_M^2]$$

Obviously, the components of field lines along $U$ are a stabilizing influence.

For the most unstable situation at the winter and summer solstices, $\psi_M$ is 78° and 102°, respectively, 78° occurring at 1630 UT and 102° occurring at 0430 UT. The inequality can then be written as
\[ U^2 > \frac{\rho_I + \rho_M}{4\pi \rho_I \rho_M} [B_I^2 \cos^2 \psi_I + 0.0431 B_M^2] \]

The most unstable configuration at the solstices nearly corresponds to the most stable configuration at the equinoxes.

The least unstable configuration during the winter and summer solstices occurs when $\psi_M$ is 55° and 125°, respectively, the former angle occurring at 0430 UT and the latter occurring at 1630 UT. The instability condition then becomes

\[ U^2 > \frac{\rho_I + \rho_M}{4\pi \rho_I \rho_M} [B_I^2 \cos^2 \psi_I + 0.329 B_M^2] \]

There are other regions where the solar streaming is perpendicular to the geomagnetic field. However, these regions are exclusively at the front of the magnetosphere, where the low value of solar streaming, as well as the absence of any seasonal dependence in this vicinity, serve to exclude it from the present discussion.

Figure 3 shows the stable and unstable regions of $U^2$ plotted against $\psi_M$ alone. The values of $\psi_M$ during the equinoxes can correspond to unstable conditions with relatively lower values of $U$ than at the solstices. If the other quantities in the inequality have no significant seasonal dependence, the instability at the flanks is dependent solely upon the angle between the dipole and the sun-earth line, as in the equinoctial hypothesis. However, any seasonal changes in the intensity of the solar streams (as suggested by the axial hypothesis) will have their Kelvin-Helmholtz effects. These have not been observed.

**Prediction of Kelvin-Helmholtz Hypothesis**

The parameters of the solar wind are known to be quite variable. These spatial variations in the solar wind are convected through the bow shock and pass through the magnetosheath. At any given time these parameters may change.
so as to alter the relative magnitudes of the left- and right-hand sides of the instability condition. When the magnetospheric term is small, the probability for instability to occur will be high. If the magnetospheric term is large, the probability for instability to occur would be correspondingly low. The times during the day of the maximum and minimum probabilities for instability are seasonally dependent and may be determined from the instability condition as in Table 1, where $D$ and $I_t$ refer to the density factor and interplanetary term, respectively. Therefore, if the instability of the magnetopause is geomagnetically effective, the Kelvin-Helmholtz explanation predicts a universal-time variation of geomagnetic activity. Illustrative plots of the probability of instability as a function of universal time are shown in Figure 4.

**TABLE 1. Diurnal Variation of Kelvin-Helmholtz Instability**

<table>
<thead>
<tr>
<th>Equinoxes</th>
<th>Summer Solstice</th>
<th>Winter Solstice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max = 1030, 2230 UT</td>
<td>Max = 0430 UT</td>
<td>Max = 1630 UT</td>
</tr>
<tr>
<td>$\psi_M = 90^\circ$, $90^\circ$</td>
<td>$\psi_M = 102^\circ$</td>
<td>$\psi_M = 78^\circ$</td>
</tr>
<tr>
<td>$U^2 &gt; D[I_t]$</td>
<td>$U^2 &gt; D[I_t] + 0.04B_M^2$</td>
<td>$U^2 &gt; D[I_t] + 0.04B_M^2$</td>
</tr>
<tr>
<td>Min = 0430, 1630 UT</td>
<td>Min = 1630 UT</td>
<td>Min = 0430 UT</td>
</tr>
<tr>
<td>$\psi_M = 78.5^\circ$, $101.5^\circ$</td>
<td>$\psi_M = 125^\circ$</td>
<td>$\psi_M = 55^\circ$</td>
</tr>
<tr>
<td>$U^2 &gt; D[I_t] + 0.04B_M^2$</td>
<td>$U^2 &gt; D[I_t] + 0.33B_M^2$</td>
<td>$U^2 &gt; D[I_t] + 0.33B_M^2$</td>
</tr>
</tbody>
</table>

$D$, density factor; $I_t$, interplanetary term.
for the December solstice, the equinoxes, and the June solstice. We have assumed here that the probability of instability is approximately a linear function of the difference between the left-hand side and the right-hand side of the instability condition. Although there is no a priori reason for a linear assumption, it represents the simplest working hypothesis. It is clear from Table 1 that the flanks have the highest probability of instability at the equinoxes and in addition have a smaller diurnal variation than the solstices. The expected diurnal

![Plot of universal-time variation of Kelvin-Helmholtz instability.](image)

Fig. 4. Plot of universal-time variation of Kelvin-Helmholtz instability.

![Predicted June minus December universal-time variation of geomagnetic disturbance compared with data analyses.](image)

Fig. 5. Predicted June minus December universal-time variation of geomagnetic disturbance compared with data analyses of (b) McIntosh [1959], (c) Mayaud [1967], and (d) Nicholson and Wulf [1955].
variation at the solstices is approximately 7 times greater than at the equinoxes, and the maximum probability of instability at the solstices is approximately equal to the minimum probability of instability at the equinoxes, as Table 1 and Figure 4 show. The ordinate of Figure 5a is the difference between the probability of instability at the June solstice and the probability of instability at the December solstice but is labeled as the difference between the expected disturbances (Δ disturbance), since the instability condition of the magnetopause is taken to be geomagnetically effective. Figure 6a presents a similar plot, but the ordinate now represents the difference between the sums of the expected disturbances at the equinoxes and the sums of the expected disturbances at the solstices. The phase and amplitude of the predicted curves of Figures 5a and 6a are confirmed by several analyses of available geomagnetic data.

**Data Analysis: Confirmation of Kelvin-Helmholtz Hypothesis**

The universal-time variation predicted by the Kelvin-Helmholtz hypothesis should be revealed in the geomagnetic data on the removal of the local-time effect. The changes of geomagnetic activity with local time are well known and have little seasonal variability. Therefore, the removal of the local-time variation may be simply accomplished by taking differences between seasons. The June minus December disturbance difference leads to the doubling of the predicted variation and is illustrated in Figure 5a. The maximum occurs at 0430 UT and the minimum at 1630 UT. The difference between March and September would

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![Figure 6](https://via.placeholder.com/150)

Fig. 6. Predicted equinoxes minus solstices universal-time variation of geomagnetic disturbance compared with data analyses of (b) McIntosh [1959], (c) Mayaud [1967], and (d) Nicholson and Wulf [1955].
obviously eliminate both the predicted effect and the local time variation. Therefore, the sum of the equinoxes minus the sum of the solstices results in a doubling of the predicted equinoctial variation seen in Figure 6a. Maximums occur at 1030 and 2230 UT and minimums at 0430 and 1630 UT.

The predicted variation is compared with several analyses of geomagnetic activity in Figures 5 and 6. Figures 5b and 6b present the indicated differences of the average daily variation of the $K$ index of 12 observatories during 1950–1955 on selected days of $Kp$ sum >20 reported by McIntosh [1959]. Using data from all stations, the average value of $K$ is obtained separately for the months of June, December, March, and September for each 3-hour interval of universal time over the indicated interval of years. The eight 3-hour intervals are then averaged for each of these months to obtain the average monthly $K$. The difference between the monthly average and the individual 3-hour monthly averages gives the 3-hour deviations from the mean for each month. This method is generally applied in determining values used in Figures 5 and 6. Figures 5c and 6c present the differences of the average daily variation of Mayaud's index ($a_m$) computed for sixteen stations for three selected years (1959, 1961, 1964), including only 3-hour intervals with $Kp$ ≥ 2 [Mayaud, 1967]. Mayaud's $a_m$ index is derived from his $K_m$ index in the same manner as $a_s$ is derived from $Kp$. The index $K_m$ is formed from the $K$ indices of ten stations in the north and six stations in the south between 45° and 55° geomagnetic latitude, after certain modifications are made so that $a_m$ is representative of an equivalent amplitude in the unit $a$. Mayaud's June solstice consists of the months May, June, July, and August; his December solstice consists of the months November, December, January, and February. Mayaud's equinoxes consist of the months March, April, September, and October. Figures 5d and 6d present the indicated differences of the average daily variation of the $K$ index for six observatories, including all days of 1940–1946 calculated by Nicholson and Wulf [1955]. These curves were formed from Nicholson and Wulf's individual monthly curves (June, December, March, and September) of the mean deviation of average $K$ values as a function of universal time. An excellent verification of the predicted phase is obtained in all three cases, and the relative amplitudes are essentially as expected. The fact that the agreement with the Mayaud data is less exact can be attributed to the longer data interval taken about the key days.

In order to verify further the Kelvin-Helmholtz mechanism, supported by the geomagnetic analysis of others, the data from four observatories closer to the auroral zone are studied: Murmansk (63.5°N, 126.2°E), Wellen (61.8°N, 237.1°E), Kerguelen (56.5°S, 127.8°E), and Maquarie Island (60.7°S, 243.0°E), where geomagnetic coordinates are indicated. The $K$ indices analyzed are taken from three weeks before to three weeks after the key dates of the equinoxes and solstices from December 1, 1959, to October 12, 1964. The indices are linearized as in the method of Bartels by forming $a_s$, the equivalent amplitude, for each station and then converting these into the range in $\gamma$. These values are averaged separately for the two stations in the north and in the south at each season for each 3-hour period of universal time. The indicated differences are then taken and plotted in Figures 7a and 7c. Composites for all four stations are shown in Figures 7b and 7d. As before, the phase verification is excellent, and, in addition, the relative amplitudes are now clearer for the near auroral stations.

Analysis of the $K$ indices from eight polar-cap observatories are performed for the same time period and, as expected, reveal no seasonal Kelvin-Helmholtz effect. It is clear that the polar cap represents quite a different regime of geomagnetic activity.

**Satellite Evidence of Waves and Instabilities**

Several satellite and related measurements are available that support the proposed instability of the magnetopause:

1. Heppner et al. [1967], with Ogo A magnetometers between ±15° geomagnetic latitude, find that the flanks of the magnetosphere are highly unstable.

2. Anderson et al. [1968] associate Imp 2 electron-flux changes having periods between 3 and 20 min with hydromagnetic disturbances propagating on the magnetopause.
3. McDiarmid and Wilson [1968] with Alouette 2 find a lowering of the high-latitude electron boundary due to a stronger coupling between the solar wind and the magnetosphere when the angle between the sun-earth line and the geomagnetic axis is near 90°.

4. Nagata et al. [1963] detect magnetically conjugate transverse giant micropulsations having left-handed polarizations at dawn and right-handed polarizations at dusk. These micropulsations (pc 5) suggest the presence of surface waves on the magnetopause [Atkinson and Watanabe, 1966].

5. Dungey and Southwood [1969] with Explorer 33 find hydromagnetic waves near the magnetopause with polarizations in agreement with data presented by Nagata et al. [1963]. They find polarizations just outside the magnetopause opposite to those inside. This evidence localizes the source of these hydromagnetic waves to the magnetopause.

6. Kaufman and Konradi [1969] with Explorer 12 find large-amplitude distortions traveling along the flanks toward the tail. These distortions are frequently triggered by a change in the properties of the solar wind.

**SUMMARY**

Kelvin-Helmholtz instability along the flanks of the magnetosphere exhibits a semiannual variation, with instability maximums at the equinoxes and instability minimums at the solstices. Kelvin-Helmholtz instabilities, generated at the magnetopause, initiate the modulation of geomagnetic disturbance detected as the semiannual variation of geomagnetic activity. The Kelvin-Helmholtz explanation predicts a universal-time variation of geomagnetic activity.
disturbance that is verified by the data. Since
the physical mechanism suggested depends on
the tilt of the dipole axis, the 'equinoctial' hy-
thesis is supported.

APPENDIX

ILLUSTRATIVE QUANTITATIVE ESTIMATES OF THE
KELVIN-HELMHOLTZ INSTABILITY CRITERION

The magnitudes of the left- and right-hand
sides of the instability criterion will be esti-
\[\text{TABLE 2. Data from Explorer 34}\]
\[
\begin{array}{|c|c|c|}
\hline
\text{Parameter} & \text{Crossing 1} & \text{Crossing 2} \\
& (K_p = 30) & (K_p = 1-) \\
\hline
B_M & 45^\circ & 30^\circ \\
\theta_M & 0^\circ & 30^\circ \\
B_t & 315^\circ & 240^\circ \\
\theta_t & 40^\circ & 15^\circ \\
\psi_t & 10^\circ & 45^\circ \\
X_{SE} & 310^\circ & 210^\circ \\
R_{SE} & 6.96 & 5.07 \\
Y_{SE} & 2.07 & 12.71 \\
Z_{SE} & -6.17 & -7.40 \\
U_r & 9.30 & 15.56 \\
U_\infty & 539 \text{ km/sec} & 382 \text{ km/sec} \\
\eta & 2.3 / \text{cm}^3 & 2.0 / \text{cm}^3 \\
\hline
\end{array}
\]
\[\text{bow shock. Both crossings were outbound, so that}
\text{the hourly average values of solar wind}
\text{speed (}U_\infty\text{) and number density (}n_\infty\text{) measured}
\text{are those obtained just after the satellite passed}
\text{through the bow shock. Table 2 lists these meas-
\text{ured parameters. The magnetic field measure-
\text{ments and satellite position (in earth radii) at}
\text{the crossing point are given in solar ecliptic}
\text{coordinates [Ness et al., 1964].}
\text{The measured parameters are not in a form}
\text{that could be used in the instability condition. In}
\text{particular, the values of the solar wind}
\text{stream speed (}U\text{) and mass densities (}\rho_\infty, \rho_r\text{) are}
\text{needed at the satellite crossing point. These}
\text{values can be obtained by using Spreiter et al.'s}
\text{[1968] gasdynamic curves (e.g., for a free-
\text{stream Mach number of 5). They display con-
tours of the ratio of magnetosheath to free-
stream values of these quantities for the dayside}
magnetosheath. The free-stream values (}U_\infty\text{ and}
\text{}n_\infty\text{) determine the stream speed and}
\text{the mass density at the crossing point. The}
magnetosphere mass density is assumed to be
\text{the same as the magnetosheath mass density in}
\text{the absence of interior plasma measurements.}
\text{The direction of the solar streaming at the}
satellite crossing point must be found so that}
\text{the components of the magnetic fields in this}
direction can be determined. The direction of}
\text{the solar streaming is uniquely determined by}
\text{the intersection of the plane containing the}
satellite crossing point and the sun-earth line
\text{and the plane formed by the magnetosheath and}
magnetospheric magnetic fields at the crossing
\text{point. The quantities }B_t \cos \psi_t \text{ and } B_M \cos \psi_M
\text{are then found.}
\text{Table 3 contains the calculated values of the}
\text{parameters to be used in the instability condi-
tion for both crossings. The instability con-}
\[
\text{TABLE 3. Calculated Values}\]
\[
\begin{array}{|c|c|c|}
\hline
\text{Quantity} & \text{Crossing 1 (}K_p = 30\text{)} & \text{Crossing 2 (}K_p = 1-\text{)} \\
\hline
U_\infty, \text{ cm/sec} & 2.16 \times 10^7 & 2.29 \times 10^7 \\
\rho_\infty = \rho_M, \text{ g/cm}^3 & 1.23 \times 10^{-23} & 7.68 \times 10^{-24} \\
B_t \cos \psi_t, \text{ gauss} & -7.09 \times 10^{-3} & 8.39 \times 10^{-4} \\
B_M \cos \psi_M, \text{ gauss} & -1.63 \times 10^{-4} & 2.12 \times 10^{-4} \\
U_r \text{, cm/sec}^2 & 4.66 \times 10^{14} & 5.25 \times 10^{14} \\
\rho_\infty + \rho_M \left( B_t^2 \cos^3 \psi_t + B_M^2 \cos^3 \psi_M \right), \text{ gauss}^2 \text{ cm}^2 / \text{g} & 4.08 \times 10^{14} & 1.08 \times 10^{14} \\
\hline
\text{Instability condition fulfilled} & \text{Yes} & \text{No} \\
\hline
\end{array}
\]
KELVIN-HELMHOLTZ INSTABILITY

dition is fulfilled for the more disturbed crossing point, but not for the quiet crossing point. Application of the gasdynamic curves of Spreiter et al. [1968] to determine conditions at the flanks yield a similar result. The two cases studied here are merely for illustration. A detailed statistical study of the Kelvin-Helmholtz inequality and its significance for the onset of geomagnetic activity is currently in progress.

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