Semiannual Variation of Geomagnetic Disturbance and Its Modulation of Shorter Period Variations

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By making use of the techniques of power spectrum analysis and simple linear filtering procedures, it is shown that the amplitude of the 27-day variation of the geomagnetic disturbance index $C_i$ is subject to a significant semiannual modulation. It is also shown that there is no amplitude modulation of this component with other periods such as the annual or 11-year period. The semiannual amplitude modulation, however, is also found in variations of $C_i$ for time periods between 27 days and 6 months, and the phase of the modulation is closely locked in phase with that of the 6-month variation of $C_i$ itself. This close agreement between the 6-month variation of $C_i$ and the 6-month amplitude modulation of such components as the 27-day variation forms the basis of a test between the two principal hypotheses that have been proposed to explain the 6-month variation of geomagnetic disturbance; namely, the so-called ‘axial’ and ‘equinoctial’ hypotheses. In the axial hypothesis, the important element is the inclination of the sun’s axis of rotation to the ecliptic plane, whereas the equinoctial hypothesis depends upon the inclination to this plane of the earth’s axis of rotation and its associated dipole magnetic field. The result of the test clearly favors the equinoctial hypothesis.

INTRODUCTION

Geomagnetic disturbance indices have long been known to contain a 27-day variation corresponding to the period of solar rotation. Recently, however, by means of a high resolution power spectrum [Shapiro and Ward, 1966], it was shown that the 27-day variation consists of three separate peaks: a central peak near 27 days and two prominent side lobes. The near-coincidence of the frequencies of the side lobes with the frequencies to be expected from an annual amplitude modulation of the 27-day variation led Coleman and Smith [1966] to conclude that the side lobes were in fact produced by just such a modulation. It was shown, however [Shapiro, 1967], that the side lobes could not be produced by an annual amplitude modulation and were therefore probably solar in origin. In the course of the analysis to test for the presence of amplitude modulation of the 27-day variation, evidence was found that was strongly suggestive of a semiannual modulation. The test used was very sensitive to the presence of amplitude modulation. Starting with daily values of the international magnetic character figure $C_i$ for the period January 1, 1884, through December 11, 1964 (29,565 values), variances over 27-day intervals were obtained, yielding a time series with 1095 values. A necessary condition for the presence of amplitude modulation of the 27-day variation is a prominent peak in the power spectrum of the time series of the variances. The power spectrum of the variances, taken from the previous report [Shapiro, 1967], is reproduced here as Figure 1. Whereas no pronounced peak is evident at a period of 1 year, there is a sharp prominent peak at $\frac{1}{2}$ year.

The existence of a peak in Figure 1 is not, however, a sufficient condition for the existence of amplitude modulation of the 27-day variation. There is a pronounced tendency for the variance of $C_i$ over some time interval to be positively correlated with the average of $C_i$ over this interval. For example, the linear correlation coefficients between the monthly mean and monthly variance, the semiannual mean and semiannual variance, and the annual mean and annual variance (using daily values of $C_i$ for the entire period January 1, 1884, through December 31, 1964) are between 0.49 and 0.55 and highly significant. The principal reason for the presence of such a correlation is that higher mean values of disturbance indices such as $C_i$ during some time period are produced primarily by an in-
crease in the number of days with high values and a decrease in the number of days with 'median' values, whereas the number of days with low values remains nearly the same. In other words there is a spread rather than a bodily shift in the frequency distribution during the period. Bartels [1963] remarked that all months, even those that are the most geomagnetically disturbed, always contain at least a few geomagnetically quiet days with extremely low values of disturbance indices. Since the spectrum of \( C_i \) does show a significant peak at \( \frac{1}{2} \) year (see, for example, Shapiro and Ward [1966]), the peak at 6 months in the spectrum of the variances (Figure 1) could indicate a 6-month amplitude modulation of a whole range of continuum frequencies, not necessarily including the 27-day variation. Consequently, an analysis was performed to remove the ambiguity concerning the question of a semiannual amplitude modulation of the 27-day variation of \( C_i \). The purpose of this note is to present the results of this analysis and discuss its implications.

**COHERENCE**

To demonstrate the nature of the relationship between the mean and variance of \( C_i \) for various time periods, the coherence spectrum [Goodman, 1957] between the mean and variance of \( C_i \) was computed for the (1085) 27-day periods mentioned above. The results are displayed in Figure 2 with the coherence as the ordinate and with frequency (per 400 \( \times \) 27 days) on a linear scale as the abscissa. The cross-correlation function was truncated at 200 lags. Thus, with 10-degrees of freedom, the solid horizontal line indicates the value of the coherence (0.63) that corresponds to the 1% level of significance. It is apparent that the mean and variance are related over a wide range of periods. No importance, however, is attached to the 'significant' values of the coherence that occur at frequencies where there is little power in the \( C_i \) spectrum. For the periods represented in Figure 2, only 11 years and 6 months have major peaks in the \( C_i \) spectrum, and both of these periods show significant coherence between the 27-day mean values and 27-day variances of \( C_i \). How-

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**Figure 1.** Power spectrum of variances over 27-day intervals of the daily index of geomagnetic disturbances \( C_i \). The total length of record consists of 1085 variances, covering a time period from January 1, 1884, through December 11, 1964. The autocorrelation function was truncated at a lag of 200, which is equivalent to 5400 days.
ever, the strength of the relationship for periods very close to 6 months is especially noteworthy. It means that the 6-month variation of $C_i$ is almost exactly related (linearly) to the 6-month oscillation in the 27-day variances. Furthermore, in as much as the spectrum of the 27-day variances (Figure 1) implies the existence of a 6-month amplitude modulation of some shorter-period variations of $C_i$, the high coherence at 6 months (Figure 2) also means that the phase of this modulation is locked in phase with that of the 6-month variation of $C_i$. It therefore seems likely that the explanation of the semiannual amplitude modulation is closely dependent upon the mechanism responsible for the 6-month variation in geomagnetic disturbance and could furnish a substantial clue to its elucidation. There is further discussion of this issue in a later section.

**ANALYSIS**

To determine whether the peak at 6 months in Figure 1 reflects a semiannual amplitude modulation of the 27-day variation of $C_i$ (as well as other frequencies) or only an amplitude modulation of other frequencies, the daily $C_i$ data were filtered to remove most of the variance associated with periods other than 27 days. The filter was constructed as follows:

We define a simple smoothing operator

$$Z_i = Z_i + 1/2S(Z_{i-1} - 2Z_i + Z_{i+1})$$  

where $Z_i$ is the discrete value of the time series $Z(t)$ at time $i$, $Z_i \mp p$ is the value of the time series at time $i \pm p\Delta t$, where $\Delta t$ is a time increment, $p$ is an integer and $S$ is a weighting element. This operator is centered, symmetrical, and does not alter the mean value of the time series.

If $Z(t)$ is expressed in terms of a series of Fourier components, it can be shown that the effect of the smoothing operator on the amplitude of any arbitrary component is given by

$$\bar{A}/A = R = 1 - 2S\sin^2(\pi f \Delta t)$$

where $\bar{A}$ and $A$ are the smoothed and unsmoothed amplitude, respectively, and $f$ is the frequency of the component. By choosing a suitable value for $S$, we can construct an operator with suitable properties. To produce the greatest damping of the high frequency components without amplifying or changing the phase of any component we must have $S = \frac{\Delta}{2}$. With this value of $S$, $R(f)$, the response function of the filter, becomes

$$R(f) = \cos^2(\pi f \Delta t)$$

Among those frequencies that can be resolved by the data ($f \leq 1$ cycle per $2\Delta t$), the minimum value of $R$ occurs at $f = \frac{\Delta}{2} \Delta t$ where it is equal to zero. The maximum value of the response function ($R = 1$) occurs at $f = 0$, which corresponds to the mean value of the time series. For frequencies greater than $\frac{\Delta}{2} \Delta t$, which might contribute to aliasing, $R(f)$ takes on values between 0 and 1. Thus with $S = \frac{\Delta}{2}$, the smoothing
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Fig. 3. Low-frequency portion of the power spectrum of the filtered, daily Ci data. The autocorrelation function was truncated at a lag of 400 days.

operator, equation 1, damps all frequencies except for the zero frequency (and even harmonics of the Nyquist frequency), it amplifies no frequencies, and it does not affect the phase of any frequency. It is also apparent from equation 3 that this operator damps high frequencies more strongly than low frequencies.

If we rewrite equation 1 with $S = \frac{1}{2}$ it becomes

$$Z_i = \frac{1}{4}(Z_{i-1} + 2Z_i + Z_{i+1})$$

(4)

If the smoothing operator, equation 4, is applied successively, $m$ times, the response function is

$$R^m(f) = \cos^m(\pi f/\Delta t)$$

(5)

$R^m(f)$ has all of the desirable properties of $R(f)$, but obviously it is more strongly damping for the higher frequencies. Furthermore, any linear combination of $R^a(f)$ and $R^b(f)$, where $a$ and $b$ are integers, would also have these same desirable properties. With appropriate choices of $a$ and $b$ one can construct suitable high pass, low pass, or bandpass filters.

If we apply equation 4 to the daily Ci data successively 100 times ($m = 100$) and subtract the resulting time series from that which is obtained by applying equation 4 50 times ($m = 50$), we are left with the filtered data used in this analysis. The response function for this filter has a maximum, equal to 0.25, at a period of 27 days. It decreases to one-half its maximum value for periods of 16 and 50 days. For periods of 9 days or less the response to the filter is less than 1% of the maximum value, and for periods of 6 months or longer the response is equal to or less than 5% of the maximum. Figure 3 shows the low-frequency portion of the power

Fig. 4. A small but typical portion of a machine plot of the filtered, daily Ci data showing the 27-day variation and the modulation of its amplitude.
spectrum of the filtered daily \(Ci\) data. It is apparent that almost all of the power of the filtered time series is concentrated around the 27-day variation.

A small but typical portion of a machine plot of the filtered \(Ci\) data is presented in Figure 4. This figure shows pronounced variations (modulation) of the amplitude of the 27-day variation of \(Ci\). Such modulation occurs throughout the entire filtered time series (January 1, 1888, through December 31, 1959), but there does not appear to be any periodicity in the modulation that can be detected by inspection. Consequently an analysis similar to that which was used to obtain Figure 1 was performed on the filtered data. Variances of the filtered daily data were computed for 30-day time intervals, but a different value was obtained at 5-day intervals. Thus a new time series was obtained whose elements consist of the variance of the filtered \(Ci\) data over 30-day intervals but with only a 5-day time spacing between consecutive elements. The use of a 30-day interval with a 5-day time spacing of the elements instead of a 27-day interval with no overlap between consecutive elements (as in Figure 1) avoids any question of aliasing of power from periods near 27 days to longer periods. Furthermore, since there is essentially no power in the filtered time series in periods shorter than 10 days (see Figure 3), the whole question of aliasing has therefore been circumvented by this procedure.

As before, the power spectrum of the new time series was obtained by means of the Fourier transform of the truncated autocorrelation function [Blackman and Tukey, 1959]. This spectrum, Figure 5, is similar in nature to Figure 1, but because it was obtained with filtered data it does not suffer from the ambiguity inherent in Figure 1. Only the low-frequency portion of the spectrum is shown in Figure 5 since the high-frequency portion is uniformly low in power. The autocorrelation function was obtained with 5254 data points (covering the length of record of the 30-day variances of the filtered data, January 1, 1888, through December 29, 1959) and was truncated at a lag of 800, which corresponds to 4000 days. The 1% significance level in Figure 5 corresponds to values that depart from the surrounding continuum by about a factor of 2. It is apparent that significant semiannual amplitude modulation of the 27-day variation of \(Ci\) is taking place. It is also evident that there is no semblance of an annual amplitude modulation. However, there appears to be a significant enhancement of power at the very low frequency end of the spectrum. This low-frequency enhancement, which also occurs in the spectrum of the 27-day variances of the unfiltered data (Figure 1) as well as in spectra of \(Ci\) itself, is produced by long-term changes in the mean and variance of \(Ci\) such as are portrayed in Table 1. These long-term changes in the mean and variance of \(Ci\) have probably been produced by changes in scaling that have taken place over the years, but they could reflect real changes taking place on a time scale too long to be resolved by the data record.

It is of interest to note from Table 1 that the

![Fig. 5. Low-frequency portion of the power spectrum of variances computed over 30-day intervals from the filtered, daily \(Ci\) data. The total length of record consists of 5254 thirty-day variances with only 5 days separating consecutive elements. The autocorrelation function was truncated at a lag of 800, which is equivalent to 4000 days.](image-url)
TABLE 1. Mean and Variance of Filtered and Unfiltered Ci Data during the First and Second 36-Year Periods

<table>
<thead>
<tr>
<th>Period</th>
<th>Unfiltered Ci Data</th>
<th>Filtered Ci Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Variance</td>
</tr>
<tr>
<td>1888-1923</td>
<td>0.622</td>
<td>0.1875</td>
</tr>
<tr>
<td>1924-1959</td>
<td>0.681</td>
<td>0.2294</td>
</tr>
</tbody>
</table>

(filtered data contain about 1% of the variance of the unfiltered data. Nevertheless the 6-month peaks in the spectra of the variances of the filtered and unfiltered data (Figures 5 and 1, respectively) are approximately equal in magnitude. The 6-month peak in Figure 1 contains about 5.2% of the total power of the time series of the variances of the unfiltered data, and the 6-month peak in Figure 5 contains about 6.4% of the total power of the time series of the variances of the filtered data. Since the unfiltered data contain much more power (variance) than the filtered data, the approximate equality of the peaks at 6 months in Figures 1 and 5 indicates the presence of semiannual amplitude modulation of most other (if not all other) frequencies corresponding to periods shorter than 6 months, in addition to that of the 27-day variation. Furthermore, it seems likely that the degree of modulation is roughly equal for all such frequencies.

There is a small peak at 11 years in the spectrum of the variances of the unfiltered data (Figure 1), but there is no comparable peak in the spectrum of the variances of the filtered data (Figure 5). Since the filtered data contain (virtually) only the 27-day variation, whereas the unfiltered data contain (in addition) a significant 6-month variation and a broad continuum, the presence of a peak at 11 years in Figure 1 implies an 11-year amplitude modulation of the 6-month variation and perhaps the nearby continuum. However, the absence of a peak at 11 years in Figure 5 shows that there is no 11 year amplitude modulation of the 27-day variation.

We summarize the results of this section with the following conclusions:

(a) Significant amplitude modulation of the 27-day variation of Ci is occurring semiannually, but with no other period.

(b) Variations of Ci over time periods between 27 days and 6 months (which contribute to the continuum in the Ci power spectrum) are also subject to semiannual amplitude modulation, roughly equal in magnitude to the amplitude modulation of the 27-day variation.

(c) There is an 11-year amplitude modulation of the 6-month variation of Ci.

THE SIX-MONTH VARIATION

The mechanism responsible for the pronounced 6-month variation in geomagnetic disturbance has been the subject of debate and controversy for many years. Two mechanisms, the so-called 'axial' and 'equinoctial,' have been proposed to account for the fact that geomagnetic disturbance tends to be high during the spring and fall and relatively low during the summer and winter. The axial hypothesis depends upon the inclination of the solar axis of rotation (7.2°) to the plane of the ecliptic, whereas the equinoctial hypothesis depends upon the inclination of the earth's axis of rotation (23.5°) to the plane of the ecliptic. Priester and Cattani [1962] have offered some recent evidence in favor of the axial hypothesis, but subsequently Bartels [1963], who had long been an exponent of the equinoctial hypothesis, presented evidence countering their argument. Since it is likely that the 6-month modulation of higher frequency geomagnetic disturbance variations is produced by the same mechanism responsible for the 6-month variation itself, it was felt that the existence of this modulation could be used as a clue to the mechanism. At least, it was felt, the modulation could provide a basis for choice between the axial and equinoctial hypotheses.

In the axial hypothesis (see, for example, Priester and Cattani [1962]), it is assumed that the solar corpuscular streams that ultimately produce geomagnetic disturbance are emitted radially from solar active regions and that they do not deviate appreciably from this radial direction at least out to the orbital distance of the earth. Furthermore, it is postulated that the cone of emission is narrow and remains restricted in the latitudinal direction but is spread out longitudinally by the rotation of the sun. Since solar active regions are generally situated...
between 10° and 20° solar latitude, the corpuscular streams from the appropriate solar hemisphere will have a higher probability of striking the earth's magnetosphere around September 7 and March 6 when the heliographic latitude of the earth obtains its maximum values (+7.2° and -7.2°, respectively). On the other hand, around June 7 and December 8, when the earth crosses the solar equatorial plane, solar corpuscular streams from both hemispheres will tend to bypass the earth.

In the equinoctial hypothesis [Bartels, 1963] it is postulated that solar corpuscular streams are emitted more or less in the plane of the ecliptic and that the inclination of the earth's axis of rotation to this plane forms the basis of the mechanism. Thus, the time of the equinoxes, when the earth's dipole magnetic field is approximately perpendicular to the oncoming solar corpuscular streams, should favor more intense geomagnetic activity.

The two hypotheses are not mutually exclusive, and the proponents of each allow the possibility that the alternative mechanism may be operating as a secondary effect. Since the phases of the 6-month variation predicted by the two hypotheses differ only by about 2 weeks and since there is appreciable 'noise' even in long-term averages of geomagnetic disturbance indices, it is not possible to differentiate conclusively between the two hypotheses solely on the basis of phase. However, the evidence on the phase of the 6-month variation does favor the equinoctial hypothesis. Smoothed values of the calendar-daily 'normals' of Ci appear to reach peak values either on or after the equinoxes [Shapiro and Ward, 1960]. The filtered Ci data (a small sample of which is shown in Figure 4), containing virtually only a 27-day variation, offers an opportunity to re-examine this question with largely 'noise-free' data.

In the axial hypothesis, higher levels of geomagnetic activity are produced by greater numbers of corpuscular streams striking the earth; and lower levels of geomagnetic activity are produced by fewer corpuscular streams striking the earth. Since there is no reason to believe that the emission of such streams from the sun is subject to a 6-month variation (there is, in fact, evidence that solar activity does not have a 6-month variation [Ward and Shapiro, 1962]), then the periods of low geomagnetic activity during summer and winter (according to the axial hypothesis) are produced by a greater than normal number of corpuscular streams bypassing the earth at these times. Thus, the axial hypothesis would predict the greatest number of geomagnetically quiet periods during the winter and summer and the greatest number of disturbed periods during spring and fall. In the equinoctial hypothesis, there is no preferential period of the year during which corpuscular streams tend to hit or bypass the earth. Thus, geomagnetically quiet periods (according to the equinoctial hypothesis) occur only when the sun is not emitting corpuscular streams. The difference between high and low levels of geomagnetic activity (say between spring and summer) is not due to a difference in the number of streams reaching the earth, but to a difference in the efficiency of the stream to produce geomagnetic disturbance. Thus, according to this hypothesis there should be a greater number of days with highly disturbed geomagnetic conditions at the time of the equinoxes and a greater number of days with moderately or weakly disturbed geomagnetic conditions during the solstices. The number of days with very quiet geomagnetic conditions, however, should be roughly equal during both the solstices and equinoxes.

The 27-day filter that was applied to the raw daily values of Ci effectively removes any long period variation of Ci such as the 6-month period and has the effect of superimposing the 27-day variation (which is superimposed in the raw data on a slowly varying continuum) onto a flat, zero-valued continuum (see Table 1). Thus, in the filtered data, geomagnetically quiet intervals during a period containing large disturbances would show up with larger negative departures from the continuum than comparably quiet intervals during a period containing only moderate or weak disturbances. In other words, if the equinoctial hypothesis is correct, we would expect to find the largest number of pronounced minima in the 27-day filtered data at about the same time of year as we find the largest number of pronounced maxima, and we would expect to find the smallest number of pronounced minima at about the same time of year as the smallest number of pronounced maxima.

The axial hypothesis, however, as indicated
Fig. 6. Frequency distribution for each day of the year of pronounced maxima in the filtered, daily $C_i$ data. The raw frequency distribution is designated as 'pass = 0.' The remaining distributions are the result of the iterative application of a low pass filter to the raw distribution, 50, 100, and 200 times.
Fig. 7. Same as Figure 6 for pronounced minima.
above, predicts the occurrence of a greater number of geomagnetically quiet periods and fewer geomagnetically disturbed periods during June and December. However, according to this hypothesis, when corpuscular streams happen to hit the earth, regardless of the time of year, they should produce geomagnetic disturbances of the same magnitude. Such a distribution of disturbed and quiet periods should produce about as large a monthly variance of $C_i$ in June and December as in March and September and because, according to this hypothesis, there should be more quiet periods in June and December, we should therefore find the greatest number of pronounced minima in the 27-day filtered data during June and December. Thus we have a clear distinction in the predicted distribution of pronounced minima and maxima of the 27-day filtered data with respect to the axial and equinoctial hypotheses. Whereas both hypotheses predict pronounced maxima occurring more often in the spring and fall, the axial hypothesis predicts the greatest number of pronounced minima in the winter and summer, and the equinoctial hypothesis predicts the greatest number in the spring and fall. In other words, if the axial hypothesis is dominant, the distributions of pronounced maxima and minima should be negatively correlated, but if the equinoctial hypothesis is dominant, the distributions of pronounced maxima and minima should be positively correlated.

We define pronounced maximum and minimum values of the 27-day filtered $C_i$ data as maxima greater than or equal to 0.10 and minima less than or equal to $-0.10$. During the period from January 1, 1888, through December 31, 1959, there were 127 days satisfying the maximum criterion and 84 days satisfying the minimum criterion. The frequency distributions of these dates, as a function of calendar date, are displayed in the bottom portions of Figures 6 and 7 and designated 'Pass = 0.' Whereas it is apparent that the frequency distributions of both the maxima and minima tend to peak in the spring and fall, because of the small number of dates spread throughout the year, the distributions tend to be ragged and noisy. One way of smoothing the distributions would be to relax the selection criteria and thereby increase the size of the sample, but this would weight the distributions with the larger number of moderate maxima and minima. Since we are interested primarily in the phases of the distributions, rather than the exact counts, a much simpler way of smoothing is to apply the operator, equation 4, to the raw frequency distributions. This operator is suitable for our purpose since it does not alter the phase of any component and selectively damp the higher frequencies that we wish to suppress. Figures 6 and 7 show the result of applying this operator to the raw distributions—both the maximum and minimum distributions—iteratively 50, 100, and 200 times. The distributions that result from 200 applications of the operator ($m = 200$ in equation 5) are sufficiently smoothed for our purpose. The amplitude of the semiannual component is reduced by only 6%, whereas the amplitude of components with periods around one month are reduced by almost 90%, and those components associated with periods of 20 days or less are reduced by more than 99%. The distribution of the maxima has peak values on March 22 and October 14. The distribution of the minima (which is not quite as smooth as the maxima) peaks on February 22 with a secondary peak on April 10 and again on October 1. The October 1 date is one-half a 27-day cycle before the October 14 date of the peak in the maxima. The April 10 date is slightly more than $1/2$ cycle after the March 22 date, and the February 22 date is 1 cycle before the March 22 date. Thus, on a time scale of six months or longer, the peaks and troughs in the frequency distributions of both pronounced maxima and minima in the 27-day filtered $C_i$ data are nearly coincident. To show the extent of this agreement, the distribution functions for the maxima and minima were linearly correlated, with the results shown in Table 2.

Table 2 shows that the 6-month variation in the distribution functions of the pronounced maxima and minima are highly positively correlated. This result is strong evidence in favor of the equinoctial hypothesis and would seem to rule out the axial hypothesis as playing any dominant role in the 6-month variation of geomagnetic activity. Whereas this conclusion is based primarily on the large positive correlation between the (200-time) smoothed distributions of pronounced maxima and minima, it is relevant to point out that the peaks in the distribution of the pronounced maxima (March
22 and October 14) occur either on or after the equinoxes and not before as would be expected from the axial hypothesis.

The principal evidence offered by Priester and Cattani [1962] in favor of the axial hypothesis involves a variation of the amplitude of the 6-month oscillation with the phase of the sunspot cycle. They used a modified ‘u’ index, which was derived by Bartels [1932] as a measure of the interdiurnal variability of the horizontal component of the geomagnetic field at the magnetic equator. They found that the 6-month variation had significantly greater relative amplitude during the descending phase of the sunspot cycle when the spot groups were situated at low solar latitudes than during the ascending phase when the spot groups are at higher solar latitudes. As we have seen above, there is evidence of an 11-year amplitude modulation of the 6-month variation of Ci. We, therefore, examined the 6-month variation of Ci as a function of the phase of the sunspot cycle to determine whether the behavior of Ci, is similar to that of the ‘u’ index. We separated the available Ci data into four parts corresponding to the minimum, ascending, maximum, or descending phases of the sunspot cycle, and determined the seasonal variation of Ci for each part separately. The results of this analysis are given in Table 3 in the form of per cent of the appropriate population values. The columns designated ‘ascending’ and ‘descending’ correspond almost exactly to Priester and Cattani’s high-latitude and low-latitude categories. There is no evidence in Table 3, however, of any significant difference in the relative amplitude of the 6-month variation of Ci as a function of the phase of the sunspot cycle. The discrepancy between the present result and that of Priester and Cattani may be due to the somewhat different data intervals used (1884–1964 in the present case and 1872–1938 in the earlier study) rather than to a real difference in the behavior of the parameters. The presence of some small systematic 11-year variation in the amplitude of the semiannual component of Ci is indicated both by the small peak at 11 years in Figure 1 and by the slight difference between the ‘minimum’ and ‘maximum’ columns of Table 3. However, it is not evident that any such variation is operating in accordance with the axial hypothesis. We, therefore, conclude that any such effect is of secondary importance and that the equinoctial mechanism is responsible for the 6-month variation of geomagnetic disturbance.

**Table 3. Seasonal Variation of Ci as a Function of the Phase of the Sunspot Cycle**

<table>
<thead>
<tr>
<th>Population Value</th>
<th>Minimum*</th>
<th>Ascending†</th>
<th>Maximum‡</th>
<th>Descending¶</th>
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<tbody>
<tr>
<td></td>
<td>0.55</td>
<td>0.63</td>
<td>0.70</td>
<td>0.70</td>
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<tr>
<td>Jan.</td>
<td>101</td>
<td>99</td>
<td>103</td>
<td>94</td>
</tr>
<tr>
<td>Feb.</td>
<td>107</td>
<td>102</td>
<td>111</td>
<td>101</td>
</tr>
<tr>
<td>March</td>
<td>115</td>
<td>114</td>
<td>109</td>
<td>111</td>
</tr>
<tr>
<td>April</td>
<td>104</td>
<td>102</td>
<td>100</td>
<td>107</td>
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<tr>
<td>May</td>
<td>95</td>
<td>98</td>
<td>99</td>
<td>101</td>
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<tr>
<td>June</td>
<td>96</td>
<td>97</td>
<td>92</td>
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<tr>
<td>July</td>
<td>90</td>
<td>93</td>
<td>98</td>
<td>91</td>
</tr>
<tr>
<td>Aug.</td>
<td>100</td>
<td>89</td>
<td>93</td>
<td>101</td>
</tr>
<tr>
<td>Sept.</td>
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<td>111</td>
</tr>
<tr>
<td>Oct.</td>
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<td>110</td>
<td>111</td>
<td>105</td>
</tr>
<tr>
<td>Nov.</td>
<td>91</td>
<td>100</td>
<td>93</td>
<td>94</td>
</tr>
<tr>
<td>Dec.</td>
<td>84</td>
<td>90</td>
<td>91</td>
<td>94</td>
</tr>
</tbody>
</table>

* 17 minimum years: 1888-1890; 1900-1902; 1911-1913; 1923; 1933-1934; 1943-1944; 1953-1954; 1964.

**Table 2. Linear Correlation Coefficients between the Distribution Functions of Pronounced Maxima and Minima as a Function of the Number of Applications of the Operator Equation 4**

<table>
<thead>
<tr>
<th>Correlation Coefficient</th>
<th>Number of Filtering Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.017</td>
<td>0</td>
</tr>
<tr>
<td>0.502</td>
<td>50</td>
</tr>
<tr>
<td>0.731</td>
<td>100</td>
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<tr>
<td>0.794</td>
<td>150</td>
</tr>
<tr>
<td>0.828</td>
<td>200</td>
</tr>
</tbody>
</table>

**References**


Shapiro, R., Interpretation of the subsidiary peaks at periods near 27 days in power spectra of geomagnetic disturbance indices, *J. Geophys. Res.*, 72, 4945, 1967.


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